Physics 507 Homework #11
Due: Thursday, Dec. 2, 2010

11.1 (a) Show that a particle under a central force with an attractive *potential* inversely proportional to the distance *squared* has a conserved quantity $D = \frac{1}{2}\vec{r}\cdot\vec{p} - Ht$.
(b) Show that the infinitesimal transformation generated by $G := \frac{1}{2}\vec{r}\cdot\vec{p}$ scales $\vec{r}$ and $\vec{p}$ by opposite infinitesimal amounts, $\vec{Q} = (1+\frac{\epsilon}{2})\vec{r}$, $\vec{P} = (1-\frac{\epsilon}{2})\vec{p}$, or for a finite transformation $\vec{Q} = \lambda\vec{r}$, $\vec{P} = \lambda^{-1}\vec{p}$. Show that if we describe the motion in terms of a scaled time $T = \lambda^2 t$, the equations of motion are invariant under this combined transformation $(\vec{r},\vec{p},t) \to (\vec{Q},\vec{P},T)$.

11.2 Consider a particle of mass m and charge q in the field of a fixed electric dipole with dipole moment[1] p. In spherical coordinates, the potential energy is given by

$$U(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{qp}{r^2}\cos\theta.$$

a) Write the Hamiltonian. It is independent of t and ϕ. As a consequence, there are two conserved quantities. What are they?
b) Find the partial differential equation in t, r, θ, and ϕ satisfied by Hamilton's principal function S, and the partial differential equation in r, θ, and ϕ satisfied by Hamilton's characteristic function W.
c) Assume W can be broken up into r-dependent, θ-dependent, and ϕ-dependent pieces:

$$W(r,\theta,\phi,P_i) = W_r(r,P_i) + W_\theta(\theta,P_i) + W_\phi(\phi,P_i).$$

Find ordinary differential equations for W_r, W_θ and W_ϕ.

[1] Please note that q and p are the charge and dipole moments here, not coordinates or momenta of the particle.