Physics 507 Homework #9
Due: Nov. 11, 2010

9.1 Two lagrangians, L_1 and L_2, which differ by a total time derivative of a function on extended configuration space,

$$L_1(\{q_i\}, \{\dot{q}_j\}, t) = L_2(\{q_i\}, \{\dot{q}_j\}, t) + \frac{d}{dt}\Phi(q_1, ..., q_n, t),$$

describe the same dynamics. That is, they give the same equations of motion $q_i(t)$, but they give differing momenta $p_i^{(1)}$ and $p_i^{(2)}$. Find the relationship between the two momenta and between the two Hamiltonians, H_1 and H_2, and show that these Hamiltonians lead to equivalent equations of motion.

9.2 A uniform static magnetic field can be described by a static vector potential $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$. A particle of mass m and charge q moves under the influence of this field.
(a) Find the Hamiltonian, using inertial cartesian coordinates.
(b) Find the Hamiltonian, using coordinates of a rotating system with angular velocity $\vec{\omega} = -q\vec{B}/2mc$.

9.3 (a) Show directly that the transformation

$$Q = \ln\left(\frac{\sin p}{q}\right), \qquad P = q \cot p$$

is canonical.

(b) Show directly that, for a arbitrary fixed constant α,

$$Q = \arctan\left(\frac{\alpha q}{p}\right), \qquad P = \frac{\alpha q^2}{2}\left(1 + \frac{p^2}{\alpha^2 q^2}\right)$$

is canonical.