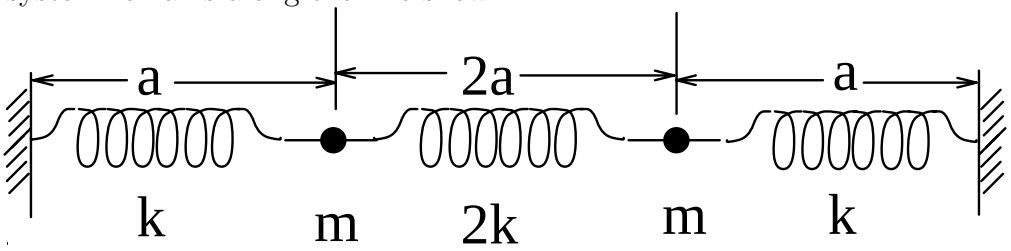
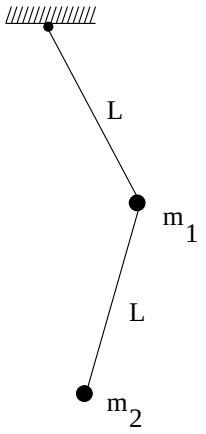
Physics 507 Homework #7
Due: Oct. 21, 2010

1.) Three springs connect two masses to each other and to immobile walls, as shown. Find the normal modes and frequencies of oscillation, assuming the system remains along the line shown.



2.)

Consider the motion, in a fixed vertical plane, of a double pendulum consisting of two masses attached to each other and to a fixed point by inextensible strings of length L. The upper mass has mass m_1 and the lower mass m_2. This is all in a laboratory with the ordinary gravitational forces near the surface of the Earth.



a) Set up the Lagrangian for the motion, assuming the strings stay taut.
b) Simplify the system under the approximation that the motion involves only small deviations from equilibrium. Put the problem in matrix form appropriate for the procedure discussed in class.
c) Find the frequencies of the normal modes of oscillation. [Hint: following exactly the steps given in class will be complex, but the analogous procedure reversing the order of U and T will work easily.]

3. [20 pts.] (a) Show that if three mutually gravitating point masses are at the vertices of an equilateral triangle which is rotating about an axis normal to the plane of the triangle and through the center of mass, at a suitable angular velocity ω, this motion satisfies the equations of motion. Thus this configuration is an equilibrium in the rotating coordinate system. Do not assume the masses

are equal.
(b) Suppose that two stars of masses M_1 and M_2 are rotating in circular orbits about their common center of mass. Consider a small mass m which is approximately in the equilibrium position described above (which is known as the L_5 point). The mass is small enough that you can ignore its effect on the two stars. Analyze the motion, considering specifically the stability of the equilibrium point as a function of the ratio of the masses of the stars.