Physics 507 Homework #2
Due: Thursday, Sept. 16, 2010

- 1.[10 pt.] Analyze the errors in the integration of Newton's Laws given in class. These were to find the change for x and p in each time interval Δt between t_i and t_{i+1} by approximating $\dot{x}(t) \approx \dot{x}(t_i)$, $\dot{p}(t) \approx F(x(t_i), v(t_i))$. Assuming F to be differentiable, show that the error which accumulates in a finite time interval T is of order $(\Delta t)^1$.

- 2.[10 pt.] Intro to computing:
  You may do this problem using any computer system and any programming language you like. You should, in any case, have gotten and found out about your account on a Physics Department Sun computer system, on which you could use python, c++ (or cc or gcc or g++), f77 (or f90), java, and probably many other options. If you need help, especially if you have never programmed, send me email. If you have a choice of programming language, tell me which.

  Write a program which uses Euler's method to integrate the motion of a harmonic oscillator with $m = 1$, $k = 1$, initial displacement 1 and velocity zero, for just over one full period. I can send you a program if you are having too much trouble writing one. You need to be able to vary the step size Δt, and extract the maximum displacement after approximately one cycle.

  Adjust the size of Δt, and see whether the error accumulated in one period meets the expectations of problem 3.

  What to turn in? The grader does not want to review your code, but he wants to see your output, or at least a selection of it with the answers, probably one page of output. Of course you should also explain what your program did.

- [more] This is optional, and will not be graded. But if you hand something in I will look at it, and hopefully be suitably impressed.

  The last two problems analyzed Euler's method for integrating an ordinary differential equation, showing the cumulative error is order Δt. There are many better methods. Find one and analyze its error, in orders of Δt, and also redo the experiment of problem 4 with this better method.

  Places you can look for better methods are textbooks on numerical analysis, or `http://netlib.bell-labs.com/netlib/ode/index.html`. One very commonly used method is called Runge-Kutta, in particular fourth order Runge-Kutta.