Classical Mechanics

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This is a preliminary version of the book, not to be considered a fully published edition.

The author welcomes corrections, comments, and criticism.

Contents

Chapter 1

Particle Kinematics

1.1 Introduction

Classical mechanics, narrowly defined, is the investigation of the motion of systems of particles in Euclidean three-dimensional space, under the influence of specified force laws, with the motion's evolution determined by Newton's second law, a second order differential equation. That is, given certain laws determining physical forces, and some boundary conditions on the positions of the particles at some particular times, the problem is to determine the positions of all the particles at all times. We will be discussing motions under specific fundamental laws of great physical importance, such as Coulomb's law for the electrostatic force between charged particles. We will also discuss laws which are less fundamental, because the motion under them can be solved explicitly, allowing them to serve as very useful models for approximations to more complicated physical situations, or as a testbed for examining concepts in an explicitly evaluatable situation. Techniques suitable for broad classes of force laws will also be developed.

The formalism of Newtonian classical mechanics, together with investigations into the appropriate force laws, provided the basic framework for physics from the time of Newton until the beginning of the last century. The systems considered had a wide range of complexity. One might consider a single particle on which the Earth's gravity acts. But one could also consider systems as the limit of an infinite number of very small particles, with displacements smoothly varying in space, which gives rise to the continuum limit. One example of this is the consideration of transverse waves on a

stretched string, in which every point on the string has an associated degree of freedom, its transverse displacement.

The scope of classical mechanics was broadened in the 19th century, in order to consider electromagnetism. Here the degrees of freedom were not just the positions in space of charged particles, but also other quantities, distributed throughout space, such as the the electric field at each point. This expansion in the type of degrees of freedom has continued, and now in fundamental physics one considers many degrees of freedom which correspond to no spatial motion, but one can still discuss the classical mechanics of such systems.

As a fundamental framework for physics, classical mechanics gave way on several fronts to more sophisticated concepts in the early 1900's. Most dramatically, quantum mechanics has changed our focus from specific solutions for the dynamical degrees of freedom as a function of time to the wave function, which determines the probabilities that a system have particular values of these degrees of freedom. Special relativity not only produced a variation of the Galilean invariance implicit in Newton's laws, but also is, at a fundamental level, at odds with the basic ingredient of classical mechanics — that one particle can exert a force on another, depending only on their simultaneous but different positions. Finally general relativity brought out the narrowness of the assumption that the coordinates of a particle are in a Euclidean space, indicating instead not only that on the largest scales these coordinates describe a curved manifold rather than a flat space, but also that this geometry is itself a dynamical field.

Indeed, most of 20th century physics goes beyond classical Newtonian mechanics in one way or another. As many readers of this book expect to become physicists working at the cutting edge of physics research, and therefore will need to go beyond classical mechanics, we begin with a few words of justification for investing effort in understanding classical mechanics.

First of all, classical mechanics is still very useful in itself, and not just for engineers. Consider the problems (scientific — not political) that NASA faces if it wants to land a rocket on a planet. This requires an accuracy of predicting the position of both planet and rocket far beyond what one gets assuming Kepler's laws, which is the motion one predicts by treating the planet as a point particle influenced only by the Newtonian gravitational field of the Sun, also treated as a point particle. NASA must consider other effects, and either demonstrate that they are ignorable or include them into the calculations. These include