Vector Identities

These are from the cover of Jackson:

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{B}\cdot\left(\vec{C}\times\vec{A}\right)=\vec{C}\cdot\left(\vec{A}\times\vec{B}\right) \quad (1)$$

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C} \quad (2)$$

$$\left(\vec{A}\times\vec{B}\right)\cdot\left(\vec{C}\times\vec{D}\right) = \left(\vec{A}\cdot\vec{C}\right)\left(\vec{B}\cdot\vec{D}\right)-\left(\vec{A}\cdot\vec{D}\right)\left(\vec{B}\cdot\vec{C}\right) \quad (3)$$

$$\vec{\nabla}\times\left(\vec{\nabla}\Phi\right) = 0 \quad (4)$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = 0 \quad (5)$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)-\nabla^2 A \quad (6)$$

$$\vec{\nabla}\cdot\left(\Phi\vec{A}\right) = \vec{A}\cdot\vec{\nabla}\Phi+\Phi\vec{\nabla}\cdot\vec{A} \quad (7)$$

$$\vec{\nabla}\times\left(\Phi\vec{A}\right) = \vec{\nabla}\Phi\times\vec{A}+\Phi\vec{\nabla}\times\vec{A} \quad (8)$$

$$\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} +\vec{A}\times\left(\vec{\nabla}\times\vec{B}\right)+\vec{B}\times\left(\vec{\nabla}\times\vec{A}\right) \quad (9)$$

$$\vec{\nabla}\cdot\left(\vec{A}\times\vec{B}\right) = \left(\vec{\nabla}\times\vec{A}\right)\cdot\vec{B}-\vec{A}\cdot\left(\vec{\nabla}\times\vec{B}\right) \quad (10)$$

$$\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right) = +\vec{A}\left(\vec{\nabla}\cdot\vec{B}\right)-\vec{B}\left(\vec{\nabla}\cdot\vec{A}\right) +\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}-\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} \quad (11)$$

With the position vector $\vec{x}$ with $r=|\vec{x}|$, $\hat{r}=\vec{r}/r$,

$$\vec{\nabla}\cdot\vec{x}=3 \qquad \vec{\nabla}\times\vec{x}=0 \quad (12)$$

$$\vec{\nabla}\cdot\left[\hat{r}f(r)\right]=\frac{2}{r}f+\frac{\partial f}{\partial r} \qquad \vec{\nabla}\times\left[\hat{r}f(r)\right]=0 \quad (13)$$

$$\left(\vec{A}\cdot\vec{\nabla}\right)\hat{r}f(r)=\frac{f(r)}{r}\left[\vec{A}-\hat{r}\left(\vec{A}\cdot\hat{r}\right)\right]+\hat{r}\left(\vec{A}\cdot\hat{r}\right)\frac{\partial f}{\partial r} \quad (14)$$

$$\vec{\nabla}\left(\vec{x}\cdot\vec{A}\right)=\vec{A}+\vec{x}\left(\vec{\nabla}\cdot\vec{A}\right)+i\left(\vec{L}\times\vec{A}\right) \quad (15)$$

$$\vec{L}=-i\left(\vec{x}\times\vec{\nabla}\right) \quad (16)$$

These are some more identities

$$\vec{\nabla}\times\left(\vec{r}\times\vec{\nabla}\right)=\vec{r}\nabla^2-\vec{\nabla}\left(1+r\frac{\partial}{\partial r}\right)$$

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}L^2$$