ansform to Frequenc

ed per solid angle, as a fu

$= |\vec{A}(t)|^2 \quad \text{where } \vec{A}(t) :=$

over time, the total energ

$$\frac{}{}\!\!\!\;^{\prime} = \int_{-\infty}^{\infty} |A(t)|^2 dt = \int_{-\infty}^{\infty}$$

g the integral over t_e, we

$$\frac{q^2}{\pi^2 c} \int_{-\infty}^{\infty} e^{i\omega(t_e + R(t_e)/c)} \left[\frac{\hat{n}}{\;} \right.$$

t there are not references

script e.

$\hat{n} \cdot \vec{r}(t)$, where observer is

egion where $\dot{\vec{\beta}} \neq 0$, which

R. Then

e

$$\frac{q^2}{\pi^2 c} e^{i\omega R/c} \int_{-\infty}^{\infty} e^{i\omega(t - \hat{n}\cdot\vec{r}(t)/}$$

eful to integrate by parts

e discuss the low frequenc

ung, that this is useful as

y parts, assuming that b

be discarded, and inserti

$$\frac{q^2\omega^2}{\pi^2 c} \left| \int_{-\infty}^{\infty} e^{i\omega(t - \hat{n}\cdot\vec{r}(t)/c)}\, \hat{n} \right.$$

We will skip pages 6

We can define the energy per unit solid angle per unit frequency,

$$\frac{d^2 I}{d\omega d\Omega} = 2|\vec{A}(\omega)|^2.$$

Our expression for the radiative part of the electric field,

$$R\,\vec{E}(t) = \frac{q}{c} \left. \frac{\hat{n} \times \left((\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}} \right)}{(1 - \hat{n}\cdot\vec{\beta})^3} \right|_{t_e}.$$

In calculating $d^2I/d\omega d\Omega$ the phase factor $e^{i\omega R/c}$ will be irrelevant.

We note that the piece in the integrand multiplying the exponential can be written as a total time derivative:

$$\frac{d}{dt}\left[\frac{\hat{n} \times (\hat{n} \times \vec{\beta})}{1 - \hat{n}\cdot\vec{\beta}} \right] = \frac{\hat{n} \times (\hat{n} \times \dot{\vec{\beta}})}{1 - \hat{n}\cdot\beta} + \frac{\hat{n} \times (\hat{n} \times \vec{\beta})(\hat{n}\cdot\dot{\vec{\beta}})}{(1 - \hat{n}\cdot\beta)^2}$$

$$= \frac{[(\hat{n}\cdot\dot{\vec{\beta}})\hat{n} - \dot{\vec{\beta}}](1 - \hat{n}\cdot\beta) + [(\hat{n}\cdot\beta)\hat{n} - \vec{\beta}](\hat{n}\cdot\dot{\vec{\beta}})}{(1 - \hat{n}\cdot\beta)^2}$$

Wigglers and Undulators

The intense peaking of forward radiation from ultrarelativistic particles, and the blue-shifting thereof, is useful for condensed matter experimentalists and biologists who could make use of very intense short pulses of X-rays. Old high-energy accelerators needn't die, they become light-sources. Monochromatic sources would also be useful.

Wigglers and Undulators use a periodic sequence of alternately directed transverse magnets to produce transverse sinusoidal oscillations, $x = a \sin 2\pi z/\lambda_0$. The angle of the beam will vary by $\psi_0 = \Delta\theta = \frac{dx}{dz} = \frac{2\pi a}{\lambda_0}$.

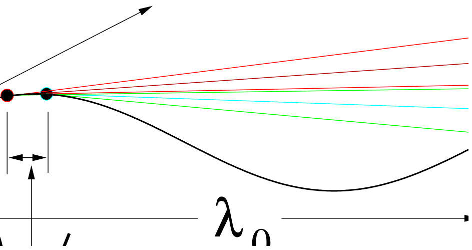
The spread in angle of the forward radiation is $\theta_r \approx 1/\gamma$, centered on the momentary direction of the beam we have a **wiggler**.

θ_r, the observer sees the s

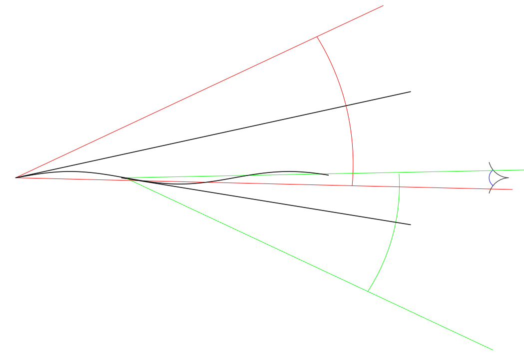
the source, that frequency

a pulse to our eye only fo

ne period, so $\Delta t_e \approx (\lambda_0/$



In the other limit, $\theta_0 \ll \theta_r$, the observer is always in the intense region of the beam, and the beam is radiating coherently. In the particle's rest frame the disturbing fields have a Fitzgerald-contracted wavelength λ_0/γ, going by at βc, so the particle sees itself oscillating at $\omega' = 2\pi c\gamma\beta/\lambda_0 \approx 2\pi c\gamma/\lambda_0$.



Thomson Scattering

er the frequency observe

$$\omega = \frac{2\omega'}{\gamma(1-\hat{n}\cdot\vec{\beta})} = \frac{4\pi}{\lambda_0(1}$$

coherent radiation, so the

to N^2 and the frequency

to $1/N$

tron has an electric field

$$\vec{E}(\vec{x},t) = \vec{\epsilon}_0 E_0 e^{i\vec{k}\cdot\vec{x}-}$$

t, it will have an accelera

$$\dot{\vec{v}}(t) = \vec{\epsilon}_0 \frac{e}{m} E_0 e^{i\vec{k}\cdot\vec{x}-}$$

is sufficiently limited to

keep the particle non-rel

$\omega^2 \ll \lambda = 2\pi c/\omega$), the tin

$\vec{\epsilon}^* \cdot \dot{\vec{v}})(\dot{\vec{v}}^* \cdot \vec{\epsilon})$ is

$$\frac{1}{2}\frac{e^2|E_0|^2}{m^2}\left|\vec{\epsilon}^*\cdot\epsilon_0\right|^2$$

$$\frac{dP}{}\Big) \quad c \quad \Big(\frac{e^2}{}\Big)^2$$

We saw (14.18) that in the particle's rest frame the electric field is given by

$$\vec{E} = \frac{q}{c^2 R}\hat{n}\times(\hat{n}\times\dot{\vec{v}}),$$

so the amplitude corresponding to a particular polarization vector $\vec{\epsilon}$ is

$$\vec{\epsilon}^*\cdot\vec{E} = \frac{q}{c^2R}\vec{\epsilon}^*\cdot\Big(\hat{n}\times(\hat{n}\times\dot{\vec{v}})\Big) = \frac{q}{c^2R}\vec{\epsilon}^*\cdot\dot{\vec{v}},$$

Dividing this by the incident energy flux $c|E_0|^2/8\pi$ we get the cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \left|\vec{\epsilon}^*\cdot\vec{\epsilon}_0\right|^2.$$

If the scattering angle is θ and the incident beam is unpolarized and the cross section summed over final polarizations, the factor of

$$\frac{1}{2}\sum_i\sum_f \left|\vec{\epsilon}_f^{\,*}\cdot\vec{\epsilon}_i\right|^2$$

$$= \frac{1}{2\pi^2}\int_0^{2\pi} d\phi_i \int_0^{2\pi} d\phi_f$$

$$\left[(\cos\theta\cos\phi_f, \sin\phi_f, -\sin\theta\cos\phi_f)\cdot(\cos\phi_i,\sin\phi_i,0)\right]^2$$

$$= \frac{1}{2\pi^2}\int_0^{2\pi} d\phi_i \int_0^{2\pi} d\phi_f \left[(\cos\theta\cos\phi_f\cos\phi_i + \sin\phi_f\sin\phi_i\right]$$

$$\frac{1}{}\left[\quad{}^2\theta + 1\right]^2$$

polarized cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)^2 \frac{1+\text{co}}{2}$$

d the **Thomson formula** ection is

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

y in parentheses is called **dius**, roughly the radius phere of charge e would $= mc^2$. (The factor of 1 arged sphere, is discarde

cross section could have been measured with an arbitrarily weak field, so recoil could be neglected, but quantum-mechanically the minimum energy hitting the electron is $\hbar\omega$, which gives a significant recoil if $\hbar\omega \approx mc^2$. In fact, if we take quantum mechanics into account we are considering Compton scattering, for which, we learned as freshman, energy and momentum conservation insure that the outgoing photon has a increased wavelength,

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta), \quad \text{or} \quad \frac{k'}{k} = \frac{1}{1+\frac{\hbar\omega}{mc^2}(1-\cos^2\theta)}.$$

It turns out that the quantum mechanical calculation (for a scalar particle) is the classical result times $(k'/k)^2$:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{QM, scalar}} = \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{k'}{k}\right)^2 |\vec{\epsilon}^{\,*}\cdot\vec{\epsilon}_0|^2 .$$