Intro to Lecture 24 Dec. 2, 2016

Last time we finished our section on integral transforms with Green's functions, both for self-adjoint 2nd order operators in one dimension and for the Poisson and Helmholtz equations in D dimensions.

Then, after a discuss with those present, we decided to scrap numerical methods and go on to relativity. Beginning with special relativity, we defined Minkowski space as a four-dimensional space with a fixed metric $\eta_{\mu\nu}$ with coordinates numbered from 0 to 3, with the zero'th coordinate being time, $x^0 = ct$.

We defined a Minkowski transformation from an unprimed chart to a primed one,

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}$$

where a^{μ} is a constant 4-vector (translation in space-time) and Λ is a Lorentz transformation matrix satisfying

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} = \eta_{\rho\sigma}.$$

We will be considering 4-vectors V^{μ} and tensors $T^{\mu\nu}$, both global variables and fields, and recall from our discussion of differential geometry that these contravariant indicies can be lowered with the metric,

$$T_{\mu}{}^{\nu} = \eta_{\mu\rho} T^{\rho\nu},$$

being carefull to keep the order unchanged, never writing T^{μ}_{ν} unless we know $T^{\mu\nu}$ to be symmetric, for otherwise $T^{\mu}{}_{\nu} \neq T_{\nu}{}^{\mu}$.

We observed that the four-momentum of a particle has $P^0 = E/c$ and $P^j = (\vec{p})_j$ for spacial j. Then Einstein has shown us that $P^2 := \eta_{\mu\nu} P^{\mu} P^{\nu} = -E^2/c^2 + \vec{p}^{\,2} = -m^2 c^2$, where m is the invariant mass of the particle, which does not change when it is moving. The equations of physics are chart-independent, if we restrict ourselves to these Minkowski-related charts.

Today we will begin with considering a collection of noninteracting point particles with charge, and discuss the charge and current densities in space-time. We will see how to express these covariantly, and what current conservation is. Then we will turn to the electromagnetic field and the stress-energy or energy-momentum tensor, both of the particles and of the fields. We will see that the conservation of energy and momentum, and of the stress-energy tensor, only holds if one includes both that of the particles and of the fields, and tells us how the fields contribute to the stress energy tensor.

This will probably be enough for today.

- Homework 10 is up and due next Monday at 5, as usual.
