Intro to Lecture 20 Nov. 16, 2016

Last time we began our discussion of the solutions to the ordinary differential equations which come from separation of variables from the Helmholtz equation and others, starting with the Bessel equation. The Bessel functions describe the radial coordinate in cylindrical coordinates. We began with the generating function $G(z,t) = \exp\{\frac{1}{2}z(t-t^{-1})\}$ which gave us $J_n(z)$, the Bessel function of the first kind of integral order n. It also gave a recursion relation and an expression for the derivative, and by generalizing these to arbitrary (nonintegral) order ν, we defined the Bessel functions $J_\nu(x)$ for arbitrary ν. These not only solve the Bessel equation but also, by a suitable singular combination of solutions, gave the irregular solutions, the Neumann functions N_n and the Hankel functions $H_\nu^{(1)}$ and $H_\nu^{(2)}$.

Today:

Finally we will discuss other related functions, the most important being the *spherical Bessel functions* j_ν and its variants, as well as the *modified Bessel functions* which are essentially Bessel functions of ix.

After finishing Bessel up, we will turn to perhaps the most important of these equations, the dependence on the polar angle in spherical coordinates. Looking first at the Poisson equation for a point charge, not at the origin, we will find the generating function for the Legendre Polynomials. We will consider dipoles and higher multipole moments as well, centered at the origin. Returning to the generating function, we will find the recursion and derivative equations, and again combining them cleverly we find Legendre's equation. It is a special case of Jacobi's, and we use Rodrigues to get the general expression, normalize it properly, and find the orthogonality integral for the Legendre polynomials. Then we will work some electrostatic problems.

If time permits, we will move on to consider P_ν for $\nu \notin \mathbb{Z}$. We will have some fun wandering in the complex plane.

- Next week contains Thanksgiving and a change in class schedule. On Wednesday, Nov. 23, we have class on the Friday schedule, at 1:40.
- Homework 9 part 1 is due Nov. 21 as usual.
- Homework 9 part 2 is a project discussing Helmholtz' equation in four-dimensional Euclidean space. This is to be done in two groups, which we need to organize. The two beautifully written reports will be due **at the start of class** at 1:40 on Nov. 23.
