Intro to Lecture 19   Nov. 11, 2016

Last time we discussed, again, the Gamma function and its relatives, concentrating on using complex analysis. We proved the Beta function is just a combination of Gammas, introduced the polygamma functions which are derivatives of $\ln\Gamma$, and used these tools to find the duplication formula and the asymptotic expansion for $\ln\Gamma(z)$ for large z. We also discussed the incomplete $\Gamma(a, x)$ and Ei, si, Ci, and li.

Today, after briefly introducing the error functions erf and erfc, important in statistics, we will turn to the solutions of the differential equations we found by separation of variables.

We begin with the Bessel equation, which describes the radial coordinate in cylindrical coordinates. But we begin not with the differential equation but with the generating function for $J_n(z)$, the Bessel functions of the first kind of integral order n. We will derive a recursion relation and an expression for the derivative, and then by generalizing these to arbitrary (nointegral) order, we will define the Bessel functions $J_\nu(x)$ for arbitrary ν, and then combine them to discuss the solutions not regular at the origin, $N_\nu(x)$ known as the Neumann functions, and the Hankel functions $H_\nu(x)$. We also have the *modified Bessel functions* which are essentially Bessel functions of ix.

With a slight modification we will find the spherical bessel functions, $j_\ell(x)$, $n_\ell(x)$, and $h_\ell^{(i)}(x)$, describing the radial dependence in spherical coordinates.

- Homework 8 is due Nov. 14 as usual.

- The following week contains Thanksgiving and a change in class schedule. On Wednesday, Nov. 23, we have class on the Friday schedule, at 1:40.

- We will also have a change in homeworks. There will be a short homework assignment #9, together with a Project 2, which will discuss Helmholtz' equation in four-dimensional Euclidean space. This is to be done in two groups, which we need to organize.