

## Intro to Lecture 18

Nov. 9, 2016

Today we will return to some important functions, beginning with the Gamma function. We will see how reframing the definition as an integral over a contour in the complex plane gives a definition good for all complex  $z$ . It encourages us to look at derivatives of  $\ln \Gamma$ , which are called the digamma and polygamma functions. We will prove the previously asserted claim that

$$B(u, v) = \Gamma(u)\Gamma(v)/\Gamma(u + v)$$

and this will lead to the *duplication formula*

$$\Gamma(2z) = \frac{2^{2z-1}}{\sqrt{\pi}} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right).$$

Using the Euler-McLaurin integration formula we will get an asymptotic expansion for  $\ln \Gamma(z+1)$  which is an improvement on Stirling's approximation. We will again touch upon the incomplete Gamma and the *exponential* and *logarithmic integral functions* as functions of a complex variable. We will also define the error functions  $\operatorname{erf}$  and  $\operatorname{erfc}$ .

Then we will turn to the functions directly involved in the solutions of the separated differential equations of physics. First we consider the Bessel functions in some detail, starting with the generating function for  $J_n(z)$ , the Bessel functions of the first kind of integral order  $n$ . We then extend the order to arbitrary values (primarily real values) and discuss the solutions not regular at the origin,  $N_\nu(x)$  known as the Neumann functions, and the Hankel functions  $H_n$ . We also have the *modified Bessel functions* which are essentially Bessel functions of  $ix$ .

- 
- Homework 8 has been posted. It is due Nov 14 as usual.
-