Intro to Lecture 14 Oct. 26, 2016

Last time we discussed branch cuts and Riemann sheets, and then turned to evaluating contour integrals by summing the residues of poles, observing that deforming the contour across a region in which the integrand is analytic does not change the integral. But that led to a discussion of what happens when there is a pole right on the integration contour, which led to defining the principal part and also the $\pm i\epsilon$ methods of making the integral well defined, and how those choices lead to differing solutions of an inhomogeneous linear differential equation.

Today, after questions, we will define the beta function by a contour integral around a cut. By deforming the contour to a circle at infinity, we will find a formula for some values $B(-\nu, 1+\nu) = -\pi/sin(\pi\nu)$ which, once we find the expression for $B(x, y)$ in terms of Γ's in Lecture K, will lead to the Euler reflection formula for the Gamma function.

Then we will return to functions having only discrete simple poles and asymptotic behavior as $z \to \infty$ bounded by $|z|^p$, and find that they can be given in terms of their residues and p terms in a Taylor series. This is called the Mittag-Leffler expansion. We will use this to consider the logarithm of an entire function with only simple zeros, and apply this to $\sin(z)/z$ to verify the product formula for the sine which we motivated in Lecture G.

Finally, we will turn to the "Method of Steepest Descents", which gives the asymptotic form of integrals $I(s) = \displaystyle\int_C g(z)e^{sf(z)}dz$ for real s large and positive. This can be used to find the behavior for large argument of many functions, including ones we will find after separating variables from Helmholtz and similar physics equations. We will also find Sterling's formula for the factorial, along with a method of getting even better approximations. We will also define and find the asymptotic behavior of the Hankel function $H_\nu^{(1)}$ which is the second solution to the Bessel equation (which blows up at $x = 0$).

After this we will turn more generally to finding solutions to the ordinary differential equations which we get from the method of separation of variables. But this will probably have to wait for next time.

Reminder: Homework #6 has been posted and is due Oct. 31.