Intro to Lecture 7 Sept 28, 2016

Last time we discussed vectors and n-forms, and more general tensor products. We defined the exterior derivative which maps n-forms into exact $(n+1)$-forms, and we found that in the association of n-forms to our customary vectors in Euclidian space shows that the exterior derivative is the gradient, the divergence, and the curl, all wrapped up as one. We also saw that $\mathbf{d}^2 = 0$, which tells us that a gradient has zero curl, and a curl has zero divergence.

Now we will discuss integration of forms and find the more sophisticated version of Stokes' theorem,

If $\mathbf{R}$ is a closed p-dimensional region of a manifold $\mathcal{M}$ with boundary $\partial\mathbf{R}$ and if $\boldsymbol{\omega}$ is a smooth $(p-1)$-form defined on $\mathbf{R}$

$$\int_{\mathbf{R}} d\boldsymbol{\omega} = \int_{\partial\mathbf{R}} \boldsymbol{\omega}.$$

For an n-dimensional manifold with metric, we will define the standard hyper-volume n-form $\boldsymbol{\Omega}$ and the covariant Levi-Civita symbol ε, and find its relation to the determinant of the metric tensor.

We will also discuss the Hodge dual, a dual relationship of r-forms on an n-dimensional manifold with metric to $(n-r)$-forms. We will find the general expression for the Laplacian.

Then we will specialize to Orthogonal Coordinates for Euclidean space. This will enable us to investigate many problems involving partial differential equations in physics. But before discussing those problems, we will make an aside to discuss how forms enlighten our relativistic understanding of electromagnetism.

Homework #3 is posted and is due next Monday, Oct. 3, as usual.

Homework #4 is going to be more involved, a project, which I would like you to do in groups. We have 8-9 people doing homeworks, seven of whom are registered. So we need 3 groups, of two or three people. Each group should have one person majoring in math (the roster says three of you are), and at least two people registered for the course. Each group will submit one homework on which the whole group agrees.

Try to get together the groups before we meet on Friday, and let me know then what the groups will be.