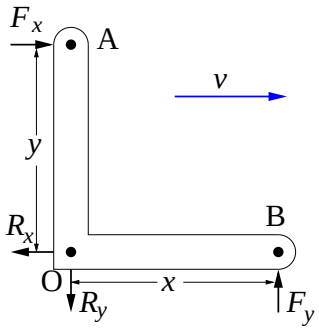
Physics 464/511 Homework #11
Due: Dec. 12, 2016 at 5:00 PM

1 Consider a rigid bracket, a carpenter's right angle "framing square" as shown. At the vertex at the origin O, a force with components $(-R_x, -R_y)$ acts, and at the ends, at A and B, forces F_x and F_y act, horizontally and vertically, respectively. In the rest frame of the bracket, the magnitudes of R_x, R_y, F_x and F_y are all equal (call that F) and the sides are also equal of length L, so $x = y = L$, so there is no net force and no net torque on the bracket, and it remains at rest.



Now consider this situation as viewed by an observer $\mathcal{O}'$ for whom the bracket is moving to the right with uniform velocity v.

(a) Describe the situation as seen by $\mathcal{O}'$, including the magnitudes and directions of each of the forces and the distances between the points O and A and between O and B. Note that it is the 4-force $f^\mu = dP^\mu/d\tau$ on a particle which transforms as 4-vector, not dP^μ/dt.

(b) What are the total force and total torque on the bracket according to $\mathcal{O}'$.

(c) Is the bracket accelerating, and is it rotating, according to $\mathcal{O}'$?

(d) What is happening to the angular momentum of the bracket, according to $\mathcal{O}'$?

If you have answered these questions correctly, the results should present a paradox. That is, they should not be in accord with naïve notions.

(e) Explain how the results can be reconciled with notions that torque gives the rate of change of angular momentum and that angular momentum is a covariant quantity that should transform suitably between different observers.

Hint: the force at A is doing positive work on the bracket, and the force R'_x is doing negative work at O. You will need to consider the energy-momentum tensor $T^{\mu\nu}(x^\alpha)$ as well as how conservation affects the total 4-momentum P^μ and the total angular momentum of the bracket.

Clearly this is a very tricky question. Those of you with poor backgrounds in special relativity will probably want to cooperate with some students with more experience. Give it a good try!