Physics 464/511 Homework #10
Due: Dec. 5, 2016 at 4:00 P. M.

1 [5 pts] A quadratically integrable function $f(x)$ is to be represented by a *finite* Fourier series. A convenient measure of the accuracy of the series is given by the integrated square of the deviation

$$\Delta_p = \int_0^{2\pi} \left[f(x) - \frac{a_0}{2} - \sum_{n=1}^{p} (a_n \cos nx + b_n \sin nx) \right]^2 dx.$$

Show that the requirement that Δ_p be minimized, that is

$$\frac{\partial \Delta_p}{\partial a_n} = 0, \qquad \frac{\partial \Delta_p}{\partial b_n} = 0, \qquad \text{for all } n$$

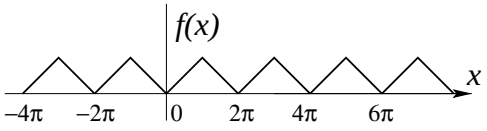
leads to choosing a_n and b_n as their exact fourier coefficients, that is, the same as the ones for $p \to \infty$.

2 [5 pts] Verify that

$$\delta(\phi_1 - \phi_2) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\phi_1 - \phi_2)}.$$

for ϕ's in the interval $(-\pi, \pi]$, or intended to be used on functions period in ϕ_2 with period 2π.

3 [5 pts] A triangular wave is represented by $f(x) = |x|$ on $x \in [-\pi, \pi]$ with f periodic with period 2π as shown. Represent $f(x)$ by a Fourier series.



4 [5 pts] A metal cylindrical tube of radius a is split lenghtwise into two non-touching halves. The top half in maintained at a potential $+V$, the bottom half at a potential $-V$, as shown. Separate the variables in Laplace's equation and solve for the electrostatic potential for $r \leq a$. Observe the resemblance between your solution for $r = a$ and the Fourier series for a square wave.