

Final exam

PHYS 677, Fall 2017

- ① Consider a Landau expansion of the free energy:

$$F = F_0 - hm + \tilde{a}_2 t m^2 + a_4 m^4$$

- a) Show that the susceptibility is given by

$$\chi = (2\tilde{a}_2 t + 12a_4 m^2)^{-1}$$

$t = \frac{T - T_c}{T_c}$ is the reduced temperature

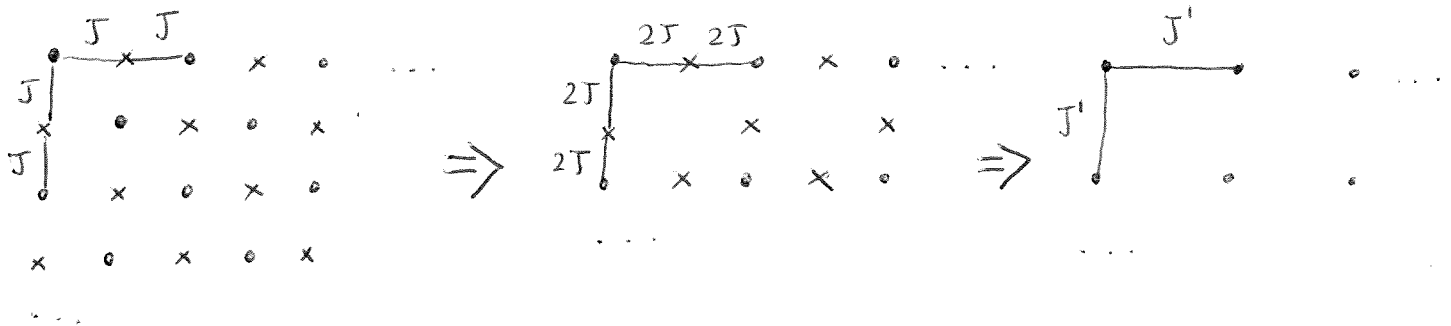
- b) Using the scaling form of m in terms of t for $t \rightarrow 0^+$ & $t \rightarrow 0^-$, show that the susceptibility exponent $\gamma_{mf} = 1$ & $\frac{\chi(t \rightarrow 0^+)}{\chi(t \rightarrow 0^-)} = 2$

- ② Write down the transfer matrix for the 1D Spin-1 Ising model in zero magnetic field:

$$\mathcal{H} = -J \sum_i S_i S_{i+1}, \quad S_i = \pm 1, 0$$

Calculate f , the free energy per spin, and show that it has expected behavior in the $T \rightarrow 0$ & $T \rightarrow \infty$ limits.

③ Consider the Migdal-Kadanoff transformation [Kadanoff *Annals of Physics* 100, 359-394 (1976)] for a 2D spin- $\frac{1}{2}$ Ising model on a square lattice:



- i) Half the bonds are removed, changing the scale of the lattice by the scaling factor $b=2$. The remaining bonds are doubled in strength.
- ii) Sites marked by a cross are removed using a 1D decimation transformation.

Show that the RG flow equation is given by

$$x' = \frac{2x^2}{1+x^4}, \text{ where}$$

$$\begin{cases} x' = e^{-2K'} \\ x = e^{-2K} \end{cases} \quad K = J/k_B T$$

Find all fixed points of this RG transform.

good luck!