INTRODUCTION TO MANY BODY PHYSICS: 620. Fall 2025

Selected Answers to Exploratory Quiz

Many thanks for spending the time to look over the quiz. It is very useful for me to have an idea of your interests and backgrounds. Here are some brief answers to some of the questions I posed.

Many thanks, Piers Coleman.

- 4. *What is the distinction between a fermion and a boson?* In three dimensions, identical particles occur in two varieties: bosons, with integral spin, and fermions, with half-integer spin. Fundamentally, bosons and fermions have different exchange statistics: when we exchange two identical particles, the wavefunction is unchanged, up to a sign. This sign is positive for bosons, but negative for fermions. This has major consequences for the physics: for fermions, no more than one particle can enter a given quantum state ("Pauli Exclusion principle"), whereas for bosons, no such restriction applies, which allows for the possibility of condensation.

- 6. *What is a path integral?* The path integral, invented by Dirac and Feynman, provides a very flexible alternative formulation of quantum mechanics that proves invaluable in many body physics and quantum field theory. Feynman and Dirac pointed out that the amplitude for a particle (or many particles) to travel along a given path $x(t)$, is given by $e^{i\mathrm{S}/\hbar}$ where S is the classical action evaluated along the path. To get the expectation value for any variable, or product of variables, we merely average the variable with this *complex* weight, over all possible paths. Such an average is called a Feynman path integral. We shall be studying this idea later in the course.

- 7. *In electromagnetism, what are the conjugate field variables analogous to position and momentum?* The vector potential is conjugate to the electric field, which plays the role of the momentum, technically, $\pi(x) = -\epsilon_0 E(x)$, so that $[\epsilon_0 E(x), A(x')] = -i\hbar\delta^D(x-x')$.

- 8. *Suppose a sudden electric field pulse is applied to a material, $E(t) = E_o\delta(t)$. Sketch, as a function of time, the current $j(t)$ that would develop in (i) a metal, (ii) a superconductor and (iii) an insulator. Please label the time-scales in your sketch, both in qualitative and quantitative terms.*

  This is a question that we will return to when we discuss metal physics, resistance and superconductivity. Here I just wanted some sketches to see how you think. The important point to realize, is that the initial response of *each* material is identical! Initially, when the field is applied, all electrons respond according to Newton's laws, so their momenta change by an amount $\Delta p = -eE_o$, giving rise to a net current $j_o = (ne^2/m)E_o$, where n is the number of electrons. The way in which this current decays is radically different in the three cases. In a metal, the current decays on a timescale which is long compared with atomic time-scales. For a mean-free-path of $l = 10,000A = 10^{-6}m$, which would be typical in say copper, since the Fermi velocity is about $v_F \sim 10^6 m/s$, this decay time is about $\tau_{tr} = l/v_F \sim 10^{-12}s$. For a superconductor, the current does not decay, even on timescales of centuries. For an insulator, the current decays on atomic timescales, of order $10^{-15}s$.

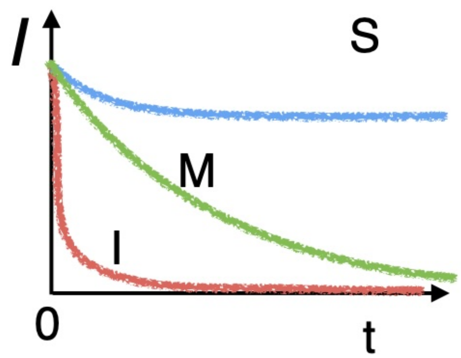

Figure 1: Decay of the current after an electric pulse in a metal (M), superconductor (S) and insulator (I).

- 9. *What happens to a fluid of bosons if you cool it to low temperatures?* The fluid undergoes "Bose-Einstein condensation". A macroscopic number of particles enter a single momentum state, and the resulting fluid develops superfluidity.

- 10. *As the complexity of a material increases, the system the system starts to develop new types of "emergent" or macroscopic property, such as magnetism. Make a list of "emergent" states of condensed matter that you know.* Here, a huge list is possible!

Emergent State/Phenomenon	Macroscopic Property.
Solid	Rigidity. (think about it)
Ferromagnet	Uniform spin polarization
Antiferromagnet	Staggered spin polarization
Superconductor	Persistent super-currrents
Electron Localization	Localized electrons-insulator
Quantum Hall Effect	Quantized Hall constant
Glass	Broken Ergodicity
Turbulence	Self-critical energy distribution
Brain	Sentient thought

- 11 *What is the Fermi energy of a metal?* The Fermi energy is the maximum energy to which states are filled inside a metal, or Fermi gas.

- 12. *A metal is cooled in a magnetic field, and becomes superconducting. What happens to the magnetic field?* Inside a superconductor, the electromagnetic field acquires a mass, and the magnetic field is excluded. ("Meissner effect").

- 13. *How does the specific heat of a metal depend on temperature?* At low temperatures, the electronic specific heat of a metal is linear in temperature. There is an additional component, produced by the phonons, which is quadratic in temperature.

- 14. *Obtain an approximate value for the integral* $I = \int_0^{5\pi} e^{-\lambda \cos^2 x} dx$*, where* λ *is very large.* This is an example of a "saddle point integral", a concept that we will later find very useful for path integrals, where it is the basis of the classical limit of quantum mechanics. Here, you can do the integral by noticing that in the vicinity of $x_o = (2n+1)\frac{\pi}{2}$, the argument of the integral is close to a Gausian $e^{-\lambda \cos^2 x} \sim e^{-\lambda (x-x_o)^2}$, with area $I_o \sim \sqrt{\pi/\lambda}$. Since there are five such peaks, the total integral is $I \approx 5\sqrt{\pi/\lambda}$. This is what we call a saddle-point approximation.

- 15. *Materials physicists can make crystals with up to 5 different elements per unit cell. How many compounds of this complexity could be made in principle?* This question can only be answered with great approximation. If we supposed that there are roughly 100 elements, which can be combined in any way we like with each other, then we'd have approximately $100^5/5! \sim 10^8$ compounds. This is certainly an underestimate, but it serves to illustrate the magnitude of diversity of properties that we expect to encounter as this new frontier is explored.