GRADUATE QUANTUM MECHANICS: 501 Fall 2001

Assignment 6. (Due Mon 20th)

Read Sakurai p. 152-168

1. Sakurai, ch 2, problem 36, p 150. An electron moves in the presence of a uniform magnetic field in the z-direction. ($\mathbf{B}=B\hat{z}$).

   (a) Evaluate

   $$[\Pi_x, \Pi_y], \tag{1}$$

   where

   $$\Pi_x = p_x - eA_x, \qquad \Pi_x = p_x - eA_x. \tag{2}$$

   (b) By comparing the Hamiltonian and the commutation relations obtained in (a) with those of the Harmonic Oscillator, show how we can immediately write the energy eigenvalues as

   $$E_{k,n} = \frac{p_z^2}{2m} + \frac{|eB|}{m}(n + \frac{1}{2}) \tag{3}$$

   where p_z is the continuous eigenvalue of the $\hat{p}_z$ operator and $n \geq 0$ is a non-negative integer.

2. (Sakurai, ch 2, problem 37, p 150.) A neutron beam is split into two components. One beam passes through a region of magnetic field pointing in the z direction, which causes the neutron spins to precess. The Hamiltonian for the neutron spin in a magnetic field is

   $$H = -g_n \frac{eB_z}{m} S_z \tag{4}$$

   where $g_n = -1.91$ is the neutron magnetic moment measured in units of $e\hbar/2m_n$. If the width of the region containing the field is l, prove that the difference in the magnetic fields that produce two successive maxima in the counting rates is given by

   $$\Delta B = \frac{4\pi h}{|eg_n|\lambda l} \tag{5}$$

   where λ is the wavelength of the neutron.