GRADUATE QUANTUM MECHANICS: 501 Fall 2001

Assignment 3. (Due Mon, Oct 15th)

Read Sakurai p 68-100.

1. Under a certain unitary transformation, the "up" and "down" states of a spin transform according to

$$\begin{aligned} |+\rangle \to U|+\rangle &= |+\rangle \cos\theta/2 + |-\rangle \sin\theta/2, \\ |-\rangle \to U|-\rangle &= |-\rangle \cos\theta/2 - |+\rangle \sin\theta/2. \end{aligned} \quad (1)$$

   (a) Calculate the matrix representation for U in terms of the $\{|\pm\rangle\}$ basis.

   (b) How do the states $|y;\pm\rangle$ transform under U?

   (c) An electron is exposed to a magnetic field $\mathbf{B} = B\hat{y}$ pointing along the y direction. The spin Hamiltonian for the electron in a magnetic field is $H = -\frac{eB}{m}S_y$. Show that the time evolution of the spin-state is described by U, where θ is time dependent. Calculate $\theta(t)$.

   (d) How long does it take to rotate an electron spin through 90^0 in a one tesla magnetic field?

2. A particle localized at x_o at $t = 0$ has wavefunction $\psi(x) = \delta(x - x_0)$.

   (a) What is the momentum space wavefunction $\phi(p,t) = \langle p|\psi(t)\rangle$ of this particle at time $t > 0$?

   (b) By transforming back to real space, show that the amplitude for a particle to go from $x_o \to x$ in time Δt is given by

$$\text{Amplitude}(x_o \to x, \Delta t) \equiv \psi(x,t) = \sqrt{\frac{m}{ih\Delta t}} \exp\left[\frac{iS}{\hbar}\right] \quad (2)$$

   where

$$S = \frac{m}{2}\left(\frac{x - x_o}{\Delta t}\right)^2 \Delta t \quad (3)$$

   What is the interpretation of S?

3. The Hamiltonian of the simple Harmonic oscillator is

$$H = \hbar\omega[a^\dagger a + \frac{1}{2}] \quad (4)$$

   where $a^\dagger$ and a are the creation and annihilation operators respectively.

   (a) Explain why $a(t) = e^{-i\omega t}a$ in the Heisenberg representation.

   (b) How do you construct the n-th excited state using the creation operator $a^\dagger$?

   (c) At $t = 0$, a system is in a linear combination of the ground-state and first excited state, $|\psi\rangle = \frac{1}{\sqrt{2}}[|0\rangle + |1\rangle]$. How does the expectation value of the position in this state vary with time?

   (d) An electron is in the ground-state of a harmonic oscillator, An experimentalist claims that after absorbing a certain number of photons he finds that the electron is never at the origin. Can you explain his experiment?