GRADUATE QUANTUM MECHANICS: 501 Fall 2001

Assignment 2. Due Oct 1

Read Sakurai, pages 37-70.

1. In a certain basis, an operator A takes the form

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{1}$$

   (a) Suppose $|-\rangle$, $|0\rangle$ and $|+\rangle$ are eigenkets of A with corresponding eigenvalues $a_- < a_o < a_+$. Write down these eigenkets in the above basis and find their corresponding eigenvalues.

   (b) A second observable B can be written

$$B = 3|-\rangle\langle -| + 2|0\rangle\langle 0| + 1|+\rangle\langle +| \tag{2}$$

   Write down B in the original basis. Are A and B compatible observables?

2. Operator A measures quantity α and operator B measures quantity β. $|a_1\rangle$ and $|a_2\rangle$ are eigenkets of A in which α attains the values a_1 and a_2. $|b_1\rangle$ and $|b_2\rangle$ are eigenkets of B in which β attains the values b_1 and b_2. Suppose

$$\begin{pmatrix} |a_1\rangle \\ |a_2\rangle \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \\ \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \end{pmatrix} \begin{pmatrix} |b_1\rangle \\ |b_2\rangle \end{pmatrix} \tag{3}$$

   α is measured and the value a_1 is obtained. β is measured, then α is measured again. What is the probability of obtaining a_1 a second time?

3. Answer the following questions about a particle moving in one dimension:

   (a) If $\psi(x) = \frac{1}{[2\pi\Delta^2]^{1/4}} e^{-x^2/4\Delta^2}$ what is the uncertainty in momentum?

   (b) If $\psi(x) = \frac{1}{\sqrt{2a}}\theta(1 - |x|)$ what is the uncertainty in momentum?

   (c) The wavefunction for a particle with a definite momentum p is $\langle x|p\rangle = (2\pi\hbar)^{-1/2} e^{ipx/\hbar}$. What is the completeness relation satisfied by the basis of states $|p\rangle$?

   (d) If $\psi(p) = \langle p|\psi\rangle$ is the wavefunction of a particle in momentum space, write down an expression for the expectation value of its position $\langle x\rangle$.

4. Use the uncertainty principle to estimate the ground-state energy of a particle of mass m, moving in a central potential $V(r) = -k/r^{3/2}$.