Lecture 26

Supersymmetry

April 19, 2007
Supersymmetry Breaking

Supersymmetry must clearly be broken since the super partners all have larger masses than the Standard Model particles.

Break the supersymmetry by introducing “soft” interactions into the Lagrangian. They are “soft” because they do no cause quadratic divergences.

The possible “soft” terms are:

- mass terms
- bi-linear mixing terms
- tri-linear scalar mixing terms
The Soft Part of the SUSY Lagrangian

For one generation

\[-L_{\text{soft}} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - B \mu \epsilon_{ij} (H_1^i H_2^j + \text{h.c.}) \]

\[+ \tilde{M}_Q^2 (\tilde{u}_L^* \tilde{u}_L + \tilde{d}_L^* \tilde{d}_L) + \tilde{M}_u^2 \tilde{u}_R^* \tilde{u}_R \]

\[+ \tilde{M}_d^2 \tilde{d}_R^* \tilde{d}_R + \tilde{M}_L (\tilde{e}_L^* \tilde{e}_L + \tilde{\nu}_L^* \tilde{\nu}_L) + \tilde{M}_e^2 \tilde{e}_R^* \tilde{e}_R \]

\[+ \frac{1}{2} \left[ M_3 \tilde{g} \tilde{g} + M_2 \tilde{\omega}_i \tilde{\omega}_i + M_1 \tilde{b} \tilde{b} \right] \]

\[+ \frac{g}{\sqrt{2} M_W} \epsilon_{ij} \left[ \frac{M_d}{\cos \beta} A_d H_1^i \tilde{Q}_j \tilde{d}_R^* \right. \]

\[+ \frac{M_u}{\sin \beta} A_u H_2^j \tilde{\tilde{Q}}^i \tilde{\tilde{u}}_R^* + \frac{M_e}{\cos \beta} A_e H_1^i \tilde{L}_i \tilde{\tilde{e}}_R^* + \text{h.c.} \]

All of the mass and interaction terms may involve 3 generational mixing matrices.

Can have well over 100 parameters!
The Higgs Sector

The scalar Higgs potential

\[ V_H = \left( |\mu|^2 + m_1^2 \right) |H_1|^2 + \left( |\mu|^2 + m_2^2 \right) |H_2|^2 \]

\[ - \mu B \left( H_1^i H_2^j + \text{h.c.} \right) + \frac{g^2 + g'^2}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 \]

\[ + \frac{1}{2} g^2 |H_1^* H_2|^2 \]

Depends only on three parameters

\[ |\mu|^2 + m_1^2 \quad |\mu|^2 + m_2^2 \quad \mu B \]

If \( \mu B \neq 0 \), the electro-weak symmetry is broken.
Electro-Weak Symmetry Breaking

If $\mu_B \neq 0$, 

$$
\langle H_1 \rangle_0 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \langle H_2 \rangle_0 = \begin{pmatrix} v_2 \\ 0 \end{pmatrix}
$$

$$
M_W^2 = \frac{g^2}{2} (v_1^2 + v_2^2)
$$

Two complex doublets have eight degrees of freedom. Three of these are gauged away to give mass to the $W^+$, $W^-$ and $Z$. There are five physical degrees of freedom left over. These result in:

- 2 charged Higgs bosons: $H^\pm$
- a $CP$-odd neutral Higgs boson: $A$
- 2 $CP$-even neutral Higgs bosons: $h$ and $H$

Once $v_1^2 + v_2^2$ is fixed by $M_W$, the Higgs sector is completely described by two independent parameters. These are generally chosen to be:

$$
\tan \beta = \frac{v_2}{v_1} \quad \text{and} \quad M_A
$$
Higgs Masses

The Higgs masses are then given by:

\[ M_A^2 = \frac{2|\mu_B|}{\sin 2\beta} \]

\[ M_{H^\pm}^2 = M_W^2 + M_A^2 \]

\[ M_{h,H}^2 = \frac{1}{2} \left\{ M_A^2 + M_Z^2 \right\} \pm \left[ (M_A^2 - M_Z^2)^2 \cos^2 2\beta + (M_A^2 + M_Z^2)^2 \sin^2 2\beta \right]^{1/2} \]

From the Higgs potential, we have the following relations among the masses

\[ M_{H^\pm} > M_W \quad M_H > M_Z \]

\[ M_h < M_A \quad M_h < M_Z |\cos 2\beta| \]

We have \[ M_h < M_Z ! \]
When we take into account one-loop corrections

\[ M_h^2 < M_Z^2 \cos^2 2\beta + \frac{3G_F}{\sqrt{2}\pi^2} m_t^4 \log \left( \frac{\tilde{m}^2}{m_t^2} \right) \]

\( M_h \) increases with increasing \( \tan \beta \) and \( M_A \) but in the MSSM its limited to be less than about 130 GeV.

![Graph showing \( M_h \) vs. \( M_A \) for different values of \( \tan \beta \).]
Higgs Couplings

Higgs couplings to fermions:

\[ \mathcal{L} = -\frac{g m_i}{2 M_W} [C_{f f h} \bar{f}_i f_i h + C_{f f H} \bar{f}_i f_i H + C_{f f A} \bar{f}_i \gamma^5 f_i A] \]

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<tr>
<th>( f )</th>
<th>( C_{f f h} )</th>
<th>( C_{f f H} )</th>
<th>( C_{f f A} )</th>
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<tr>
<td>( u )</td>
<td>( \cos \alpha )</td>
<td>( \sin \alpha )</td>
<td>( \cot \beta )</td>
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<td>( d )</td>
<td>( \frac{-\sin \alpha}{\cos \beta} )</td>
<td>( \frac{\cos \alpha}{\cos \beta} )</td>
<td>( \tan \beta )</td>
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\( C_{f f h} \to 1 \) yields Standard Model coupling.

Higgs Couplings to \( u, c, t \) in SUSY

Higgs Couplings to \( b \) in SUSY
Higgs Couplings (cont.)

Higgs couplings to gauge bosons:

\[ ZZ_h \sim \frac{ig M_Z}{\cos \theta_W} \sin(\beta - \alpha) \]

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\[ WW_h \sim ig M_W \sin(\beta - \alpha) \]

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\[ ZhA \sim \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} \]

\[ ZHA \sim -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} \]
\[ \tan 2\alpha = \frac{(M_A^2 + M_Z^2) \sin 2\beta}{(M_A^2 - M_Z^2) \cos 2\beta} \]

As \( M_A \rightarrow \infty \)

\[ \sin(\beta - \alpha) \rightarrow 1 \quad \text{and} \quad \cos(\beta - \alpha) \rightarrow 0 \]

In the limit of large \( M_A \), the \( H \) Higgs boson decouples from the gauge bosons while the \( h \) Higgs boson has Standard Model couplings. In the limit of large \( M_A \), it will be very difficult to differentiate a SUSY Higgs sector from the Standard Model Higgs boson.
Stop Mixing

The scalar partners of the left and right handed fermions mix to form mass eigenstates. Mixing will be largest for the stop since the mixing is proportion to the quark mass.

If the SUSY breaking scale is much greater that $M_Z$, $m_t$, and $A_t$, all of the squarks will be degenerate.

If the SUSY breaking is on the scale of the electro-weak breaking scale, then the mixing becomes large and the stop will become the lightest squark.
Charginos and Neutralinos

Charginos:

The charged winos, $\tilde{w}^\pm$, and the charged higgsinos, $\tilde{h}^\pm$, mix forming two charginos, $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^\pm$.

Neutralinos:

The bino, $\tilde{b}$, the neutral wino, $\tilde{w}^3$, and the neutral higgsinos, $\tilde{h}_1^0$ and $\tilde{h}_2^0$ mix forming four neutralinos, $\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$ and $\tilde{\chi}_4^0$.

The lightest neutralino is usually assumed to be the LSP.
Running Coupling Constants

The gauge coupling constant run.

\[ \alpha_i(Q^2) = \frac{\alpha_i(M^2)}{1 + \frac{b\alpha_i(M^2)}{\pi} \ln \left( \frac{M^2}{Q^2} \right)} \]

**Standard Model**

- \( b_1 = \frac{4}{3} N_{\text{gen}} + \frac{N_H}{10} \)
- \( b_2 = -\frac{22}{3} + \frac{4}{3} N_{\text{gen}} + \frac{N_H}{6} \)
- \( b_3 = -11 + \frac{4}{3} N_{\text{gen}} \)

**SUSY**

- \( b_1 = 2N_{\text{gen}} + \frac{3N_H}{10} \)
- \( b_2 = -6 + 2N_{\text{gen}} + \frac{N_h}{2} \)
- \( b_3 = -9 + 2N_{\text{gen}} \)
With SUSY, the gauge coupling constants are unified at $M_X \approx 10^{16}$ GeV.

$$\sqrt{\frac{5}{3}} g_1(M_X) = g_2(M_X) = g_3(M_X) = g_X$$

Gaugino masses also assumed to unify

$M_i(M_X) = m_{1/2}$

With masses running as

$$M_i(M_W) = m_{1/2} \frac{g^2_i(M_W)}{g^2_X}$$
Common scalar mass also assumed

\[ m_1^2(M_X) = m_2^2(M_X) = \tilde{M}_Q^2(M_X) = \tilde{M}_d^2(M_X) \]
\[ = \tilde{M}_u^2(M_X) = \tilde{M}_L^2(M_X) = \tilde{M}_e^2(M_X) = m_0 \]

As well as common \( A \) parameters

\[ A_t(M_X) = A_b(M_X) = \cdots = A_0 \]

With these assumptions the SUSY sector is completely described by 5 parameters at the GUT scale.

- a common scalar mass: \( m_0 \)
- a common gaugino mass: \( m_{1/2} \)
- a common trilinear coupling: \( A_0 \)
- a Higgs mass parameter: \( \mu \)
- a Higgs mixing parameter: \( B \)

This set of assumptions is called SUGRA. Although what if anything it has to do with gravity is unclear.
The SUGRA model is beloved by phenomenologists because it reduces the hundred odd SUSY parameters to 5. There is little justification for SUGRA. The SUGRA parameters are:

\[ m_0, \quad m_{1/2}, \quad A_0, \quad \mu, \quad B \]

The \( Z \) mass constrains \(|\mu B|\) leaving \( \text{sign}(\mu) \) as a free parameter and usually the \( B \) parameter is traded for \( \tan \beta \). So the parameters that most people talk about are:

\[ m_0, \quad m_{1/2}, \quad A_0, \quad \text{sign}(\mu), \quad \tan \beta \]

In general, squark masses are larger than slepton masses because their driven by their strong interactions.

\[ \tilde{m}_L^2(M_W) \sim \tilde{m}_e^2(M_W) \sim \tilde{m}_0^2 \]

\[ \tilde{m}_q^2(M_W) \sim m_0^2 + 4m_{1/2}^2 \]
SUGRA Electro-Weak Symmetry Breaking

In Standard Model, electro-weak symmetry results when the $\mu^2$ parameter in the Higgs potential is negative. [Note: the SM $\mu$ parameter is a totally different object from the $\mu$ parameter of SUSY.]

The question arises: Why is $\mu^2$ negative?

The SUGRA model naturally accounts for $\mu^2 < 0$ if the top mass is large.
Running of the Masses

Masses as well as charges run.

\[ \frac{dM_h^2}{d \ln(Q^2)} = \frac{3\lambda_T^2}{16\pi^2}(\tilde{M}_{QL}^2 + \tilde{M}_{tR}^2 + M_h^2 + A_t^2) \]

\[ \frac{d\tilde{M}_{tR}^2}{d \ln(Q^2)} = -\frac{4\alpha_3}{3\pi} m_t^2 + \frac{2\lambda_T^2}{16\pi^2}(\tilde{M}_{QL}^2 + \tilde{M}_{tR}^2 + M_h^2 + A_t^2) \]

\[ \frac{d\tilde{M}_{QL}^2}{d \ln(Q^2)} = -\frac{4\alpha_3}{3\pi} m_t^2 + \frac{\lambda_T^2}{16\pi^2}(\tilde{M}_{QL}^2 + \tilde{M}_{tR}^2 + M_h^2 + A_t^2) \]

\( M_h^2 \) runs more strongly than \( \tilde{M}_{tR}^2 \) or \( \tilde{M}_{QL}^2 \) and can become negative.

For large \( \lambda_t \) (which is proportional to \( m_t \)) we have:

\[ M_h^2(Q^2) = M_h^2(M_X^2) \]

\[ -\frac{3}{8\pi^2} \lambda_t^2(\tilde{M}_{QL}^2 + \tilde{M}_{tR}^2 + M_h^2 + A_t^2) \ln(M_X^2/Q^2) \]
Running of the Masses (cont.)

Sample MSSM Mass Spectrum

Figure 9: Sample masses of SUSY particles in a SUSY GUT. At the GUT scale $M_X$, we have taken $m_0 = 200$ GeV, $m_{1/2} = 100$ GeV, $\mu = 100$ GeV and $A_t = 0$. The solid line is the lightest neutral Higgs boson mass. The dashed lines are the gaugino masses (the largest is the gluino) and the dot-dashed lines are typical squark masses.

If $m_t$ is large enough $M_h(M_W^2)$ goes negative. But $m_t$ can’t be too large or the squark masses will also go negative.

$$m_t = 175 \text{ GeV} \text{ is just right!}$$
$b - \tau$ Unification

Theorist like to have

$$\lambda_b(M_X^2) = \lambda_\tau(M_X^2)$$

$$\Rightarrow \tan \beta \sim \frac{m_t}{m_b} = 35$$

This has important phenomenological consequences. The couplings of the lightest Higgs boson to $b$ quarks and $\tau$’s is enhanced relative to the Standard Model.