Lecture 23

CP Violation

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Case of Pure Mass Term

\[ H = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \]

\[ M_{11} = \langle K^0 | H | K^0 \rangle \quad M_{12} = \langle K^0 | H | \bar{K}^0 \rangle \]

\[ M_{21} = \langle \bar{K}^0 | H | K^0 \rangle \quad M_{22} = \langle \bar{K}^0 | H | \bar{K}^0 \rangle \]

CPT invariance \( \Rightarrow \quad M_{11} = M_{22} \)

CP invariance \( \Rightarrow \quad M_{12} = M_{21} \)

\( H \) is Hermitian \( \quad H = H^\dagger \)

\[ \Rightarrow \quad M_{21} = M_{12}^* \]

CP symmetry violated if \( M_{12} \) is complex
General Case with Particle Decay

Need to take into account decay $K^0$ and $\bar{K}^0$.

$$H = \begin{pmatrix}
M_{11} - i \frac{\Gamma_{11}}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\
M_{21} - i \frac{\Gamma_{21}}{2} & M_{22} - i \frac{\Gamma_{22}}{2}
\end{pmatrix}$$

Where $M$ and $\Gamma$ are Hermitian $M^\dagger = M \quad \Gamma^\dagger = \Gamma$

Assuming $CPT$ invariance $M_{11} = M_{22} \quad \Gamma_{11} = \Gamma_{22}$

$$H = \begin{pmatrix}
M_{11} - i \frac{\Gamma_{11}}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\
M_{12}^* - i \frac{\Gamma_{12}^*}{2} & M_{11} - i \frac{\Gamma_{11}}{2}
\end{pmatrix}$$

$CPT$ is violated if either $M$ or $\Gamma$ is complex
For generic pair of neutral mesons \( P^0 \) and \( \bar{P}^0 \) mass eigenstates are

\[
|P_A\rangle = p |P^0\rangle + q |\bar{P}^0\rangle \quad |P_B\rangle = p |P^0\rangle - q |\bar{P}^0\rangle
\]

where \( |p|^2 + |q|^2 = 1 \)

\[
\left(\frac{q}{p}\right)^2 = \frac{H_{21}}{H_{12}} = \frac{M_{12}^* - i \Gamma_{12}/2}{M_{12} - i \Gamma_{12}/2}
\]

\[
|P_{A,B}(t)\rangle = e^{-i(M_{A,B}-i \Gamma_{A,B}/2)t} |P_{A,B}(0)\rangle
\]

\[
M_{A,B} = M_{11} \pm \text{Re}\left(\sqrt{H_{12}H_{21}}\right) = M_{11} \pm \text{Re}F
\]

\[
\Gamma_{A,B} = \Gamma_{11} \pm \text{Im}\left(\sqrt{H_{12}H_{21}}\right) = \Gamma_{11} \pm 2 \text{Im}F
\]

\[
F = \sqrt{(M_{12} - i \Gamma_{12}/2)(M_{12}^* - i \Gamma_{12}/2)}
\]

If \( CP \) invariant \( F = M_{12} - i \frac{\Gamma_{12}}{2} \)
$K^0$, $\bar{K}^0$ System

$q$ and $p$ deviate only slightly from 1

\[
\frac{q}{p} = \frac{1 - \epsilon}{1 + \epsilon}, \quad p = \frac{1 + \epsilon}{\sqrt{2}}, \quad q = \frac{1 - \epsilon}{\sqrt{2}}
\]

\[
|K_L\rangle = \frac{1}{\sqrt{2}} \left( (1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle \right)
\]

\[
= \frac{1}{\sqrt{1 + |\epsilon|^2}} \left( |K_2\rangle + \epsilon|K_1\rangle \right)
\]

\[
|K_S\rangle = \frac{1}{\sqrt{2}} \left( (1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle \right)
\]

\[
= \frac{1}{\sqrt{1 + |\epsilon|^2}} \left( |K_1\rangle + \epsilon|K_2\rangle \right)
\]

\[
\frac{A(K_L \rightarrow 2\pi)}{A(K_S \rightarrow 2\pi)} = \epsilon
\]
Measurement of $\epsilon$

Start with a pure $K^0$ at $t = 0$

$$p + n \rightarrow p + \Lambda^0 + K^0$$

$$|K(t = 0)\rangle = |K^0\rangle = \frac{1}{2p}(|K_S\rangle + |K_L\rangle)$$

$$|K(t)\rangle = \frac{1}{2p}\left(e^{-(\Gamma_S/2+im_S)t}|K_S\rangle + e^{-(\Gamma_L/2+im_L)t}|K_L\rangle\right)$$

$$\langle K_1|K(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-(\Gamma_S/2+im_S)t} + \epsilon e^{-(\Gamma_L/2+im_L)t}\right)$$

$$I(K_1) = |\langle K_1|K(t)\rangle|^2 = \frac{1}{2}\left[e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t}
+ \epsilon \left(e^{i(m_L-m_S)t} + e^{-i(m_L-m_S)t}\right)\right]$$

$$I_{K \rightarrow 2\pi}(t) = I_{K \rightarrow 2\pi}(0)$$

$$\times \left[e^{-\Gamma_S t} + |\epsilon|^2 e^{-\Gamma_L t} + 2|\epsilon|e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m t + \phi)\right]$$
Measurement of $\epsilon$ (cont.)

\[
\frac{\text{Rate}(K_L \rightarrow e^+\nu_e\pi^-) - \text{Rate}(K_L \rightarrow e^-\bar{\nu}_e\pi^+)}{\text{Rate}(K_L \rightarrow e^+\nu_e\pi^-) + \text{Rate}(K_L \rightarrow e^-\bar{\nu}_e\pi^+)}
\]

\[
= \frac{|\langle K^0 | K_L \rangle|^2 - |\langle \bar{K}^0 | K_L \rangle|^2}{|\langle K^0 | K_L \rangle|^2 + |\langle \bar{K}^0 | K_L \rangle|^2} = \frac{1 + \epsilon^2 - |1 - \epsilon|^2}{1 + \epsilon^2 + |1 - \epsilon|^2}
\]

\[
= \frac{1 - \left|\frac{q}{p}\right|^2}{1 + \left|\frac{q}{p}\right|^2} = 2 \text{Re}(\epsilon)
\]

\[
\epsilon = e^{i\phi} \quad |\epsilon| = 2.3 \times 10^{-3} \quad \phi = 44^\circ
\]

$\epsilon$ is small because $V_{td}$ and $V_{ts}$ are both small.
Superweak Model

All of the $CP$ violation that we’ve discussed so far comes from $K^0 - \bar{K}^0$ mixing.

$K_L$ decays to $2\pi$ because it is not a $CP$ eigenstate but contains a small admixture of the $CP$ even state $K_1$.

$$|K_L\rangle = \frac{1}{\sqrt{2}} \left( (1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle \right)$$

If all $CP$ violation comes from mixing, we can’t distinguish Standard Model $CP$ violation resulting from the complex phase of $V_{td}$ from that of a simple “toy” superweak model on which some new $CP$-violating ‘superweak” force causes mixing between $K^0$ and $\bar{K}^0$.

We need another measure of $CP$ violation to rule out the superweak toy model.
Direct $CP$ Violation

Look for $CP$ violation in decay (not mixing)

\[
\frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon' \quad \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon'
\]

\[
\frac{|A(K_L \rightarrow \pi^+\pi^-)|^2}{|A(K_L \rightarrow \pi^0\pi^0)|^2} = 1 + 6 \frac{\epsilon'}{\epsilon}
\]

\[
\frac{\epsilon'}{\epsilon} = 2.2 \times 10^{-3} \quad \text{very small}
\]

Non zero values rules out superweak model.
Since $B^0$ and $\bar{B}^0$ have few common decay modes

$$\Gamma_{12} \ll M_{12} \quad \Rightarrow \quad \left| \frac{q}{p} \right| = 1,$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{td}}{V_{td}^*} = e^{-2i\beta}$$

$$\tan \beta = \frac{\eta}{1 - \rho} \quad \tan \beta \approx 0.43$$

$CP$ violation in mixing much larger than for $K^0, \bar{K}^0$ system.

\[ B^0 \] \hspace{1cm} \[ W \] \hspace{1cm} \[ W \] \hspace{1cm} \[ \bar{B}^0 \]

\[ \sim V_{tb}V_{td} \quad V_{tb} \text{ is large} \]
$B^0, \bar{B}^0$ System (cont.)

$CP$ violation in the $B^0-\bar{B}^0$ system is large because the phase difference between $q$ and $p$ is large.

However $\epsilon = \frac{1}{\sqrt{2}}(p - q)$ is small so we can’t see $CP$ in the same way as for the kaon system.

Also, since $\Gamma_1 \approx \Gamma_2$, the lifetime difference between $B_L$ and $B_S$ is very small and we can’t produce a beam of $B_L$ to measure and we can $K_L$.

Instead we need to focus of on common decay modes of $B^0$ and $\bar{B}^0$. That is

$$B^0 \to f \quad \bar{B}^0 \to f$$

where $f$ is a final state to which both $B^0$ and $\bar{B}^0$ can decay.
**CP Violation in the B System**

Let $A_f$ be the amplitude for $B^0 \rightarrow f$ and $\bar{A}_f$ be the amplitude for $\bar{B}^0 \rightarrow f$.

Start with a pure $B^0$ at $t = 0$

$$|B(t = 0)\rangle = |B^0\rangle = \frac{1}{\sqrt{2}}(|B_S\rangle + |B_L\rangle)$$

$$|B(t)\rangle = \frac{1}{\sqrt{2}}\left( e^{-(\Gamma_S/2+im_S)t}|B_S\rangle + e^{-(\Gamma_L/2+im_L)t}|B_L\rangle \right)$$

Taking $\Gamma = \Gamma_1 \approx \Gamma_2$ and $m = m_1 \approx m_2$

$$|B(t)\rangle = \frac{e^{-(\Gamma/2+im)t}}{2} \left[ (e^{-i\Delta mt/2} + e^{i\Delta mt/2})|B^0\rangle + \frac{q}{p} \left( e^{-i\Delta mt/2} - e^{i\Delta mt/2} \right)|\bar{B}^0\rangle \right]$$
**CP Violation in the $B$ System**

\[
A_{B \rightarrow f}(t) = e^{-(\Gamma/2+im)t} A_f \left[ \left( e^{-i\Delta mt/2} + e^{i\Delta mt/2} \right) \frac{2}{2} + \frac{q \bar{A}_f}{p A_f} \left( e^{-i\Delta mt/2} - e^{i\Delta mt/2} \right) \right]
\]

\[
= e^{-(\Gamma/2+im)t} A_f \left[ \cos(\Delta mt/2) - \frac{q \bar{A}_f}{p A_f} i \sin(\Delta mt/2) \right]
\]

If $f$ is a $CP$ eigenstate, then $A_f = \pm \bar{A}_f$ depending on whether $f$ is even or odd $CP$.

\[
|A_{B \rightarrow f}(t)|^2 = |A_f|^2 e^{-\Gamma t} \left[ 1 \mp \sin 2\beta \sin(\Delta mt) \right]
\]

\[
|A_{\bar{B} \rightarrow f}(t)|^2 = |\bar{A}_f|^2 e^{-\Gamma t} \left[ 1 \mp \sin 2\beta \sin(\Delta mt) \right]
\]
\[ |A_{B \rightarrow f}(t)|^2 = |A_f|^2 e^{-\Gamma t} [1 \mp \sin 2\beta \sin(\Delta mt)] \]

\[ |A_{\bar{B} \rightarrow f}(t)|^2 = |\bar{A}_f|^2 e^{-\Gamma t} [1 \pm \sin 2\beta \sin(\Delta mt)] \]

Tag particle as \( B^0 \) or \( \bar{B}^0 \) at \( t = 0 \).

Plot following ratio as a function of \( t \)

\[ \frac{|A_{B \rightarrow f}|^2 - |A_{\bar{B} \rightarrow f}|^2}{|A_{B \rightarrow f}|^2 + |A_{\bar{B} \rightarrow f}|^2} = \mp \sin 2\beta \sin(\Delta mt) \]

Ratio will oscillate with frequency \( \frac{\Delta m}{2\pi} \).

Amplitude of oscillation give \( \sin 2\beta \).
Unitarity Triangle

Unitarity condition: \( V^\dagger V = 1 \)

\[ \Rightarrow \quad (3\text{rd column}) \times (1\text{st column}) = 0 \]

\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\( \tilde{\rho} = \rho \left( 1 - \frac{\lambda^2}{2} \right) \)

\( \tilde{\eta} = \eta \left( 1 - \frac{\lambda^2}{2} \right) \)

\( \eta \) gives height of the triangle.

Without \( CP \) violation \( \eta = 0 \), the triangle collapses to a line.

Constrain Standard Model by measuring angles and length of sides of triangle.
Unitarity Triangle Confronts Data

Figure 11.2: Constraints from the text on the position of the apex, A, of the unitarity triangle. The apex is found at \( \delta = 60^\circ \pm 14^\circ \), \( \alpha = 23.7^\circ \pm 2.1^\circ \), and \( \beta = 23.7^\circ \pm 2.1^\circ \). The invariant measure of CP violation is \( J = (2.88 \pm 0.33) \times 10^{-5} \).
Neutrino Oscillations

Within the past few years neutrino oscillations have been discovered.

\[\nu_e \leftrightarrow \nu_\mu \quad \nu_\mu \leftrightarrow \nu_\tau\]

This means that neutrinos have mass and that there is a four parameter (3 real and 1 complex phase) matrix describing flavor-mass mixing in the lepton sector.

This is only now being sorted out. It will take many years of very difficult experiments to determine the parameters of the lepton mixing matrix.

A prime objective will be to determine the amount of \(CP\) violation in the lepton sector. Maybe it is large but the experiments to measure it are extremely difficult.

Why is this important?
Matter Dominated Universe

Why do we live in a matter dominated universe?

Big Bang predicts:

\[
\frac{N_B}{N_\gamma} = \frac{N_{\bar{B}}}{N_\gamma} = 10^{-18}
\]

We observe:

\[
\frac{N_B}{N_\gamma} = 10^{-9} \quad \frac{N_{\bar{B}}}{N_\gamma} < 10^{-13}
\]
Sakharov Conditions

Sakharov presented three feature required to have a matter dominated universe.

1. There must be baryon number violating interactions. Quarks must couple to leptons. I.e. protons decay. This is interactions are present in many extensions of the Standard Model.

2. For the matter dominance transition to have occurred the universe must have been in a non equilibrium state. This holds for the inflationary big bang scenario.

3. There must be $CP$ violation. The complex phase in the quark mixing matrix provides $CP$ viloation in the Standard Model but it isn’t nearly large enough to do the job.

The extra $CP$ violation required must either come from physics beyond the Standard Model or maybe the $CP$ violation in the lepton sector is large enough.