Lecture 1

An Introduction to Quantum Mechanics

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Mechanics

Mechanics is a theory that describes how particles respond to interactions.

The interactions themselves are given by some other theory.

- electromagnetism
- gravity
- etc.
Two Pillars of Physics

Special Relativity  The Stage
• 4-dimensional space-time
• invariance of physics under Lorentz transformations

Quantum Mechanics  The Language
• specification of the state of a system
• description of how a system responds to interactions (dynamics)

Merging of special relativity and quantum mechanics
Relativistic Quantum Field Theory

Two other important aspects of physics. Not fully established yet.

Fundamental Constituents  The Actors
• what are the basic elements of the universe
• what are their properties

Fundamental Interactions  The Plot
• how do the basic elements interact with each other
Classical Mechanics (1687 – 1897)

Space and Time
• space is the stage. time is a parameter
• example: \( x(t) \)

Classical Particles
• point particles
• intrinsic properties: mass, electric charge

Newtonian Mechanics

\[
F = m \frac{d^2x}{dt^2} = m \ddot{x}
\]

\[
\left[ \text{for conservative forces} \quad F = -\frac{dV}{dx} \right]
\]

second-order differential equation \( \Rightarrow \) state of system completely specified by:

\[
x(t_0) \quad \dot{x}(t_0)
\]

or equivalently by:

\[
x(t_0) \quad p(t_0)
\]
Particle Trajectory

Given $F(x, t)$ can solve differential equation either analytically (in a few cases) or numerically.

Position and velocity are continuous functions of time

\[
x(t + dt) = x(t) + \dot{x}(t)dt
\]

\[
\dot{x}(t + dt) = \dot{x}(t) + \ddot{x}(t)dt
\]

\[
\ddot{x}(t) = \frac{F(x, t)}{m} dt
\]

Once the state of a system is specified at some time $t_0$, its state at any other time $t$ is completely determined
Failure of Classical Physics

Newtonian physics seems intuitive but it is completely wrong.

– cannot independently specify position and momentum
– outcomes of experiments not fully determined but only given in terms of probability
– particle does not follow a trajectory

Partial list of classical physics failures

blackbody radiation
photoelectric effect
electron diffraction
specific heat of gases
atomic spectra
stability of atoms
The Double Slit Experiment

- source of electrons with well defined energy located a large distance from a wall with two slits
- electrons of well defined momentum incident uniformly on wall
- plot distribution of electron on detection wall located some distance from the slit wall

Interference
With both slits open get interference pattern for arrival of electrons at detection wall.

Probability
Although all electrons are identical, cannot specify where a given electron will be detected, only described in terms of a probability distribution.

No trajectory
Electron does not go through one slit or the other. Result with both slits open is not the sum of the results with each slit open individually
Formulation of Quantum Mechanics

• State completely specified by a complex function of either $x$ or $p$ (not both).

$$\psi(x) \quad \text{or} \quad \tilde{\psi}(p)$$

⇒ No Trajectory

• A general state is a linear superposition of states

$$\psi(x) = \psi_1(x) + \psi_2(x) + \cdots$$

⇒ Interference

• Probability density for finding the particle at position $x$ or with momentum $p$ is given by:

$$|\psi(x)|^2 \quad \text{or} \quad |\tilde{\psi}(p)|^2$$

Normalization:

$$\int |\psi(x)|^2 \, dx = 1 \quad \int |\tilde{\psi}(p)|^2 \, dp = 1$$

⇒ Probability

Overall complex phase of state has no physical significance but relative phase of superposed states is important. It gives interference.
let $\psi(x) = \psi_1(x) + \psi_2(x)$

with $\psi_1(x) = A_1 e^{i\delta_1}$, $\psi_2(x) = A_2 e^{i\delta_2}$

$$|\psi(x)|^2 = |A_1 e^{i\delta_1} + A_2 e^{i\delta_2}|^2$$

$$= A_1^2 + A_2^2 + A_1 A_2 \left( e^{i(\delta_1 - \delta_2)} + e^{-i(\delta_1 - \delta_2)} \right)$$

$$= A_1^2 + A_2^2 + 2A_1 A_2 \cos(\delta_1 - \delta_2)$$

Euler Relation:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The relative phase, $\delta_1 - \delta_2$, between two states is physically significant.
Probability

Classical Physics

We use probability to make predictions in classical physics when we don’t have full information about the system. For example, the probability that a particular molecule of air will have a certain velocity or the probability that it will rain tomorrow. In a classical world if we completely knew the state of the system, we could make an exact prediction for the velocity of the molecule or whether we should bring an umbrella.

Quantum Mechanics

Probability is fundamental to the formulation of quantum mechanics. Even if we have complete knowledge of the state of a system, we cannot make exact predictions. For example, if we have a set of identical muons sitting on the table, we cannot say exactly when any individual muon will decay only that there is an exponential probability distribution for the decay with mean decay time of $1.2 \, \mu s$. 
Ensembles

• Outcomes of single experiments cannot be predicted. We, instead, must imagine a large set of identical measurements carried out on identically prepared systems (ensembles). As the number of measurements $N \to \infty$ quantum mechanics tells us the fraction that will have a particular outcome. For any finite number of measurements the fraction will only be approximately equal to the prediction.
Measurement

It is important to understand the concept of measurement in quantum mechanics and how it differs from classical physics.

- In classical physics, the position and momentum of a particle exists independently of any measurement. In quantum mechanics, the position and momentum of a particle only have physical meaning after they are measured.

- In classical physics, an ideal measurement does not affect the state of the system. In quantum mechanics, in general, a measurement changes the state of the system. It changes $\psi(x)$.

- In quantum mechanics, we can only measure the position of a particle or its momentum. We cannot measure both simultaneously.

- We will sometimes speak of an ideal measurement that precisely determines the position or precisely determines the momentum. In practice this isn’t possible, rather the position or momentum can only be determined within some finite region. In reality, a precise position or a precise momentum for a particle may not even be possible. We’ll discuss this shortly.
Non-Local Effect of Measurement

Now for the hard one.

- A measurement has a non-local effect. It will instantaneously change the wave function everywhere even half way across the universe. Many people don’t like this but it doesn’t matter. That’s the way things are.

For example, a particle may have a wave function that extends from here to the Moon. That means that in a measurement of its position, it has some probability to be on the Moon and some probability to be in front of me. If I measure it to be on the table in front of me, the probability of it being on the Moon instantaneously goes to zero.
Relation between $\psi(x)$ and $\tilde{\psi}(p)$

Since $\psi(x)$ and $\phi(p)$ each contain full information on the state of the system, we must be able to write one in terms of the other. Only possibility is:

$$\tilde{\psi}(p) = \int_{-\infty}^{\infty} f(x, p) \psi(x) \, dx$$

$$\psi(x) = \int_{-\infty}^{\infty} g(x, p) \tilde{\psi}(p) \, dp$$

We can derive what the functions $f(x, p)$ and $g(x, p)$ must be by assuming how $\psi(x)$ and $\tilde{\psi}(p)$ should transform under translations in space.