1. Consider the step potential

\[ V(x) = \begin{cases} 0 & \text{if } x < 0 \\ V_0 & \text{if } x > 0 \end{cases} \]

For a free particle fired from left with \( E < V_0 \)

(a) Solve the time-independent Schrödinger Eq. as we did in class, and sketch \( |\Psi(x,t)|^2 \) as a function of \( x \).

(b) Express \( R \) and \( T \) as a function of \( E \).

(c) What values does \( \frac{|\Psi(x=0,t)|^2}{|A|^2} \) approach for \( \frac{E}{V_0} \ll 1 \) and for \( \frac{E}{V_0} \approx 1 \), respectively?

Here, "A" represents the amplitude of the incoming wave.

2. Now consider

\[ V(x) = \begin{cases} V_0 & \text{if } x < 0 \\ 0 & \text{if } x > 0 \end{cases} \]

A free particle is again fired from left with \( E > V_0 \)

Do (a) and (b) of Prob. 1 for this problem as well.
5. A free particle has the initial wave function

\[ \psi(x,0) = \begin{cases} \frac{1}{\sqrt{2a}}, & \text{if } -a < x < a \\ 0, & \text{otherwise} \end{cases} \]

(a) Rewrite this function using \(\Theta(x-a)\) and \(\Theta(x+a)\)

(b) Using \(\frac{d\Theta(x)}{dx} = \delta(x)\) and \(\int_{-\infty}^{\infty} \delta(x) f(x) dx = f(0)\)

find the expectation values \(\langle p \rangle\) and \(\langle p^2 \rangle\) at \(t=0\)

(c) Find the expectation values \(\langle x \rangle\) and \(\langle x^2 \rangle\) at \(t=0\)

(d) Check the uncertainty principle