Problem Set 4, Due November 15th 2000

Problem 1:

\[ H_0 = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} \]

\[ N = \sum_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} \]

\[ |\psi_0\rangle = \prod_{k<k_F} c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger |0\rangle \]

\[ \rho_q \equiv \sum_{k\sigma} c_{k+q\sigma}^\dagger c_{k\sigma} \]

\[ \Delta_q^\dagger \equiv \sum_{k} c_{-k+q\uparrow}^\dagger c_{k\downarrow}^\dagger \]

a) zero temperature.
Calculate the Fourier transform of \( G_\rho(t,q), G_p(t,q) \) in time.

\[ G_\rho(t,q) = \langle T(\rho_q(t)\rho_{-q}(0)) \rangle \]

\[ G_p(t,q) = \langle T(\Delta_q(t)\Delta_q^\dagger(0)) \rangle \]

write an expression it will involve a \( k \) - integration consider the case of \( d \) dimensions can you evaluate some of the integrals in \( d=1,2,3 \)?

Problem 2 Repeat the first problem at finite temperature Finite temperature - if \( \tau \) is the imaginary time it is convenient to use Fourier series

\[ G_\rho(i\nu_n,q) = \int_0^\beta e^{i\nu_n\tau} G_\rho(\tau,q) d\tau \]

\[ G_p(i\nu_n,q) = \int_0^\beta e^{i\nu_n\tau} G_p(\tau,q) d\tau. \]

Evaluate them in 3 and 2 dimensions.

Now \( G_\rho \) and \( G_p \) are defined by

\[ G_\rho(\tau,q) = \frac{1}{Z} \text{tr} e^{-\beta(H_0-\mu N)} \rho_q(\tau)\rho_{-q}(0) \]
\[ G_\rho(\tau, q) = \frac{1}{Z} tr e^{-\beta(H_{o-N})} \Delta_q(\tau) \Delta^\dagger q(0) \]

Problem 3: Evaluate the following Matsubara sums

\[ T \sum_n e^{i\omega_n \beta} \] and \[ T \sum_n e^{i\omega_n \beta}. \]

Interpret the result why do the two sums differ