We are going to use the mathematica program which we constructed last week to practice the evaluation of several many body observables.

So take the one dimensional Hubbard model with 4 sites and periodic boundary conditions, at half filling (2 up particles and 2 down particles).

Let us evaluate the following quantities for $t=1$, and $U=4$ and $8$ and plot them vs $U$.

a) kinetic energy per site b) interaction energy per site c) $\frac{1}{N_s} \sum_i < S_i.S_{i+1}>$, average spin exchange energy.

Can you explain qualitatively the trends you see in your results?