Physics 417: Problem Set 1 (Due in class Wednesday 9/18)

Problem 1: Blackbody radiation

Recall the Planck blackbody spectrum: \( \rho(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \).

(a) What is the frequency \( \nu \) at which the spectrum peaks as a function of the blackbody temperature \( T \)? (An approximate numerical formula is okay.)

(b) At what ranges of temperatures does the blackbody peak in the visible spectrum (which you can take to be the range of wavelengths 400 – 700 nm)?

Problem 2: Atoms

(a) Estimate the lifetime of hydrogen in the classical nuclear model, i.e. an electron orbiting a proton bound by the Coulomb force. (Hint: use the Larmor formula for the power radiated by an accelerating charge, \( \frac{dE}{dt} = \frac{2}{3} \frac{q^2 a^2 c^3}{}\).)

(b) Estimate the size of an atom using Avogadro’s number, the molar mass and the typical density of matter (say ~ 10 g/mol and ~ 1 g/cm³).

(c) In class we neglected the gravitational force between the proton and electron in describing the nuclear model. Justify why this is correct.

Problem 3: Magnetic moments

(a) Consider a solid spherical ball of mass \( m \) rotating about the \( z \) axis with a charge \( q \) uniformly distributed on its surface. Show that the magnetic moment \( \vec{\mu} \) is related to the angular momentum \( \vec{L} \) by

\[
\vec{\mu} = \frac{5q}{6m} \vec{L}
\]

(b) Find the analogous relation for a solid spherical ball of mass \( M \) with charge \( Q \) uniformly distributed throughout the ball. What is the proportionality factor in this case?

(c) Convert your answers in (a) and (b) into g-factors. Do either of them exceed the g-factor for the electron spin, i.e. \( g \approx 2 \)? Comment on what it would take to achieve \( g = 2 \) while maintaining spherical symmetry.
Problem 4: Spin states

In class, we will show that the spin operators \( S_x, S_y, S_z \) can be represented as

\[
S_x = \frac{\hbar}{2}(\langle -| + | + \rangle - | + \rangle \langle - |), \quad S_y = \frac{i\hbar}{2}(\langle -| + | - \rangle + | + \rangle \langle - |), \quad S_z = \frac{\hbar}{2}(| + \rangle \langle + | - | - \rangle \langle - |)
\] (2)

(a) In the basis where \( |+\rangle \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( |-\rangle \rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} \), find the matrix representation of \( S_{x,y,z} \).

(b) Show that \([S_i, S_j] = i\epsilon_{ijk} h S_k\) and \(\{S_i, S_j\} = \frac{\hbar^2}{2}\delta_{ij}\), using both the operator representation (2) and the matrix representation.

Problem 5: More on states and operators

Suppose I have a 3-state Hilbert space spanned by the orthonormal states \( |\alpha\rangle, |\beta\rangle, |\gamma\rangle \), and an operator \( A \) that acts as:

\[
A|\alpha\rangle = \frac{1}{\sqrt{2}}|\beta\rangle, \quad A|\beta\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |\gamma\rangle), \quad A|\gamma\rangle = \frac{1}{\sqrt{2}}|\beta\rangle
\] (3)

(a) Find a matrix representation of this operator \( A \).

(b) What are the eigenvalues and the normalized eigenkets of \( A \)? Can you guess a physical system which is described by all this?

Problem 6: GRE quickies

I realize that many of you will have to take the GRE this semester. I would like to help you study for the quantum mechanics portion of the test, so periodically I will give you HW problems taken from past GRE exams. They are not intended to take very long. (On the actual test you have about 3 hours to solve about 100 problems.)

(a) (from 1996) When alpha particles are directed onto atoms in a thin metal foil, some make very close collisions with the nuclei of the atoms and are scattered at large angles. If an alpha particle with an initial kinetic energy of 5 MeV happens to be scattered through an angle of 180°, what is the distance of closest approach to the scattering nucleus? (Assume that the metal foil is made of silver, with \( Z = 50 \).)

(b) (from 2001)

\[
|\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle \\
|\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle
\] (4)
The states $|1⟩$, $|2⟩$, $|3⟩$ are orthonormal. For what values of $x$ are the states $|ψ_1⟩$, $|ψ_2⟩$ given above orthogonal?

(c) (from 2001): The state $|ψ⟩ = \frac{1}{\sqrt{6}}|−1⟩ + \frac{1}{\sqrt{2}}|1⟩ + \frac{1}{\sqrt{3}}|2⟩$ is a linear combination of three orthonormal eigenstates of the operator $A$ corresponding to eigenvalues $−1$, $1$ and $2$. What is the expectation value of $A$ in this state?