1. We can use Eqns. 7.47 and 7.48 with $\lambda = 2\pi$ and $k = 2\pi/\lambda = 1$. Then using standard integral tables,

$$A_m = (1/\pi) \int_0^{2\pi} \theta^2 \cos(m\theta) \, d\theta = 4/m^2,$$

and

$$B_m = (1/\pi) \int_0^{2\pi} \theta^2 \sin(m\theta) \, d\theta = -4\pi/m.$$

(Note that most terms either vanish or cancel at the two limits.) This shows that $f(\theta)$ has the form given. If one keeps only the first two terms in the sum ($m = 1$ and $m = 2$) one has the approximate expression

$$f^2(\theta) = 4\pi^2/3 + 4 \cos(\theta) - 4\pi \sin(\theta) + \cos(2\theta) - 2\pi \sin(2\theta).$$

This is shown along with the exact periodic expression $f(\theta)$ in the accompanying figure.

2 (Prob. 7.45) Here $\Delta l_c = 20\lambda_0 = c\Delta t_c$, and therefore the bandwidth is

$$\Delta \nu = 1/\Delta t_c = c/(20\lambda_0) = 3 \times 10^8/(20 \times 500 \times 10^{-9} m) = 3 \times 10^{13}\text{ Hz}.$$  

3. (Prob. 8.5) The propagation vector has magnitude $k$ and equal $x$ and $y$ components, so $k = k(i + j)/\sqrt{2}$. The polarization vector is perpendicular to this and also in the $x$-$y$ plane, and real since the polarization is linear. Thus $E_0 = E_0(i - j)/\sqrt{2}$. The electric field in this wave is then given by

$$E = E_0 \sin(k \cdot r - \omega t).$$

4. (Prob. 8.17) The flux density after the new middle polarizer is $I' = I_1 \cos^2(45^\circ) = I_1/2$ and the light is polarized at $45^\circ$ to both the vertical and horizontal directions. After the final polarizer the intensity is $I_2 = I' \cos^2(45^\circ) = I'/2 = I_1/4$, instead of $I_2 = 0$ without the middle polarizer.
For the middle polarizer at $\theta$ to the vertical, and thus at $90^\circ - \theta$ to the horizontal, $I' = I_1 \cos^2(\theta)$ and

$$I_2 = I' \cos^2(90^\circ - \theta) = I' \sin^2(\theta) = I_1 \cos^2(\theta) \sin^2(\theta) = I_1 \sin^2(2\theta)/4.$$  

(The last step from the trignometric identity $\sin(2\theta) = 2 \sin \theta \cos \theta$. ) Clearly this has its maximum possible value of $I_1/4$ when $2\theta = 90^\circ$ or $\theta = 45^\circ$.

5. (Prob. 8.32 ) Snell’s Law gives $\sin \theta_i = \sin \theta_i / n_{ti} = \sin 40^\circ / 1.5 = 0.428$ or $\theta_i = 25.4^\circ$. The reflectances for the two polarizations can be calculated from Eqns. 8.26 and 8.27, which result from the Fresnel Equations derived in Chapt. 4:

$$R_\parallel = (\tan(\theta_i - \theta_i)/ \tan(\theta_i + \theta_i))^2 = (\tan(14.6^\circ)/ \tan(65.4^\circ))^2 = 0.0142$$

and

$$R_\perp = (\sin(\theta_i - \theta_i)/ \sin(\theta_i + \theta_i))^2 = (\sin(14.6^\circ)/ \sin(65.4^\circ))^2 = 0.0768.$$  

There is therefore more light polarized perpendicular to the plane of incidence than polararized in the plane of incidence in the reflected beam, so the reflected light is not “natural” or unpolarized. Using the formula from Born and Wolf for the degree of polarization one finds

$$|(R_\parallel - R_\perp)/(R_\parallel + R_\perp)| = |(0.0142 - 0.0768)/(0.0142 + 0.0768)| = 0.688.$$