1. (Prob. 3.17) The two fields can be written as

\[ E = E_0 \cos(k \cdot r - \omega t), \]

and

\[ B = B_0 \cos(k \cdot r - \omega t), \]

where \( k \) is the propagation vector and \( \omega = kc \) is the angular frequency. For the wave to propagate in the x-y plane at 45° to the x axis we must have

\[ k = k(i + j)/\sqrt{2}. \]

\( E_0 \) must be perpendicular to \( k \), so if it is in the x-y plane we must have

\[ E_0 = E_0(i + j)/\sqrt{2}, \]

with \( E_0 = 10V/m \). (The negative of this would also be possible.) \( B_0 \) must be perpendicular to both \( k \) and \( E_0 \) and thus in the \( \pm z \) direction. To decide which one requires that the Poynting vector, which is in the \( E_0 \times B_0 \) direction, be parallel to \( k \), which requires \( B_0 \) to be in the +z direction. (See Prob. 3.3 for another approach.) The magnitude \( B_0 = E_0/c \) as required by Maxwell’s equations. The flux density (time average of the Poynting vector) is

\[ I = c\varepsilon_0 E_0^2/2 = 3 \times 10^8 m/s \times 8.854 \times 10^{-12} C^2/(Nm^2) \times (10V/m)^2/2 = 0.13 W/m^2. \]

(To see that the units come out right note that a volt times a Coulomb is a Joule.)

2 (Prob. 3.24) (a) The flashlight emits light energy at the rate of \( 3.0V \times 0.25A \times 0.01 = 7.5 mW \). Each photon has an energy of \( hc/\lambda \), so the number of photons emitted per second by the flashlight is

\[ 7.5 \times 10^{-3} J/s/(6.626 \times 10^{-34} Js \times 3 \times 10^8 m/s/550 \times 10^{-9} m) = 2.1 \times 10^{16} \text{photons/s}. \]

(b) In one second these photons fill a volume of area \( 10 cm^2 \) and \( 3 \times 10^8 m \) long, so the number of photons per volume is

\[ 2.1 \times 10^{16}/(10 \times 10^{-4} m^2 \times 3 \times 10^8 m) = 6.9 \times 10^{10} \text{photons/m}^3. \]
The number of photons in one meter of the beam is this number times the area \(10^{-3} \text{m}^2\) of the beam, or \(6.9 \times 10^7\) photons.

(c) The flux density of the beam is just

\[
I = 7.5 \times 10^{-3} \text{W}/10 \times 10^{-4} \text{m}^2 = 7.5 \text{W}/\text{m}^2.
\]

3. (Prob. 3.31) (a) Assuming perfect reflection from the metal (giving an extra factor of two) the pressure due to the sunlight is

\[
\mathcal{P} = 2 < S > /2 = 2 \times 1.4 \times 10^3 \text{W}/\text{m}^2 / 3 \times 10^8 \text{m}/\text{s} = 9.33 \times 10^{-6} \text{N}/\text{m}^2.
\]

(b) The average value of the Poynting vector at the surface of the sun will be greater by a factor of \((1.5 \times 10^{11} \text{m}/0.7 \times 10^9 \text{m})^2 = 4.6 \times 10^4\) because the same energy is spread over a spherical area proportional to the square of the distance from the center of the sun. If there is again complete reflection the radiation pressure at the surface of the sun is \(0.43 \text{N}/\text{m}^2\), whereas for complete absorption the pressure would be half this or \(0.21 \text{N}/\text{m}^2\).

4. For a single resonant frequency \(n = (1 + x)^{1/2}\) where \(x = Nq_x^2/|2\epsilon_0(\omega_0^2 - \omega^2)|\) is the contribution due to a charged oscillator. For low enough density \(N\), \(x\) becomes small compared to one and the binomial expansion \((1 + x)^{1/2} \approx 1 + (1/2)x + \ldots\) gives the required result. Note that if this expression is valid then the index of refraction must be very close to one. Examples are the gases in Table 3.2 on p. 66 of Hecht.

5. Looking at the curves and formulas in Sec. 3.5.1 one sees that the index of refraction increases rapidly to a very large value near the resonant frequency. For lead glass, with the resonant frequency in the near UV, close to the visible, this rapid increase will begin in the visible, with the index for violet light considerably larger than that for red. For fused silica the resonant frequency is in the far UV, far from the visible, so the index of refraction will increase only slowly with frequency across the visible.