1. Suppose the electron were a spin $3/2$ particle obeying Fermi-Dirac statistics.
   (a) Write the configuration of a hypothetical $Ne$ ($Z=10$) atom made up of such "electrons". (That is, the $S = 3/2$ analog of $(1s)^2(2s)^2(2p)^6$). Show that the configuration is highly degenerate.
   (b) If exchange and spin-orbit splitting are taken into account, what ground-state configuration is selected? Please write your answer in spectroscopic notation? ($^{2S+1}L_J$, where $S$, $L$ and $J$ stand for the total spin, the total orbital angular momentum, and the total angular momentum respectively.)

2. Consider a particle in one dimension moving under the influence of some time-independent potential. The energy levels and the corresponding eigenfunctions for this problem are assumed to be known. We now subject the particle to a pulse travelling at speed $c$, represented by a time-dependent potential, $V(t) = A\delta(x - ct)$
   (a) Suppose at $t = -\infty$ the particle is known to be in the ground-state whose energy eigenfunction is $\langle x|i \rangle = u_i(x)$. Obtain the probability for finding the system in some excited state with energy eigenfunction $u_f(x) = \langle x|f \rangle$ at $t = +\infty$.
   (b) Interpret your result in (a) physically by regarding the delta function pulse as a superposition of harmonic perturbations; recall
   \[ \delta(x - ct) = \frac{1}{2\pi c} \int_{-\infty}^{\infty} d\omega e^{i\omega(x/c - t)} \]
   Emphasize the role played by energy conservation, which holds even quantum mechanically as long as the perturbation has been on for a very long time.

3. A rotator whose orientation is specified by the angular co-ordinates $\theta$ and $\phi$ rotates in a potential that favors orientation in the $y-z$ plane. The Hamiltonian is given by
   \[ H = AL^2 + Bh^2 \cos 2\phi \]
   with $A >> B$.
   (a) Calculate the $s$ ($l=0$), $p$($l=1$) and $d$($l=2$) energy levels of this system in first order perturbation theory.
   (b) Write down the leading order expressions for the corresponding energy eigenkets.

4. (a) Verify that, outside the range of a short-range potential, the wave-function
   \[ u(r, \theta) = \frac{1}{r} \left( 1 + \frac{i}{kr} \right) e^{ikr} \cos \theta \]
   represents an outgoing p-wave.
   (b) A beam of particles represented by the plane wave $e^{ikz}$ is scattered by an impenetrable sphere of radius $a$, where $ka << 1$. Considering only $s$ and $p$ components, and working to order $(ka)^2$, obtain an expression for the differential scattering cross-section for scattering at an angle $\theta$. 


5. The Dirac equation for a relativistic fermion is taken to be

\[
\frac{i\hbar}{\partial t} \partial \psi = \left[ -i c \bar{\alpha} \cdot \vec{\nabla} + \beta mc^2 \right] \psi
\]  

(3)

where

\[
\bar{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
\]

are the \(4 \times 4\) Dirac matrices and \(\psi\) is a four-component spinor.

(a) Derive the properly normalized four component wave-function for a positive energy spin-up particle travelling in the z-direction with momentum \(p\). You may use the notation \(E(p) = \sqrt{m^2 c^4 + p^2 c^2}\). Please normalize your wavefunction to one, assuming that the wave is travelling in a box of side-length 1.

(b) Derive the normalized four-component wave-function for a negative energy spin-up particle travelling in the z-direction with momentum \(p\).

(c) Suppose that a particle has wavefunction at time \(t = 0\)

\[
\psi(\vec{x}, 0) = \frac{1}{L^{3/2}} \begin{pmatrix} \alpha \\ 0 \\ \beta \\ 0 \end{pmatrix} e^{i \vec{p} \cdot \vec{x}/\hbar}
\]

where \(\vec{p} = (0, 0, p)\) is along the z-axis. Calculate the form of the wavefunction at later times and give the probabilities \(p^\pm\) for the particle to be in a positive, or negative energy state.