ELECTRIC AND MAGNETIC FORCES

About this lab

With the discovery that electric charge comes in discrete particles with inertial mass, it became of great interest and practical importance to elucidate the mechanisms of interaction among the particles. Two forces were found, electric and magnetic, the latter affecting only moving charges. The field concept was invented to describe the spatial aspects of charge particle interactions.

Mastery of electricity has enabled modern technological life in all aspects. Examples in which the motion of charged particles is directly affected by application of electric or magnetic fields produced by other charges include cathode ray tubes (e.g. monitors) and mass spectrometers (e.g. chemical analysis).

There are only two fundamental equations below:

The electrical force law

> Equation 1a \( \vec{F}_E = q \vec{E} \)

and its electric potential correlate

Equation 1b \( \frac{1}{2}mv^2 = q \Delta V \), and

The magnetic force law

> Equation 2 \( \vec{F}_B = q \vec{v} \times \vec{B} \)

where \( \vec{v} \) is the particle's velocity vector.

All the other equations derive from these, as applied to the particular geometries of the apparatus (in simplifying approximations): Plane parallel electrodes, and parallel circular current coils in Helmholtz configuration. Since the physical arrangement deviates from the idealizations in our equations, your experimental results will deviate from the predictions from those idealizations. Nevertheless, as various ratios will show, agreement is not bad.

Appendix 1 shows simulations of beam paths in constant \( \vec{E} \), \( \vec{B} \), and \( \vec{E} \times \vec{B} \) fields.

Appendices 2 and 3 give procedure and detailed derivations.

References: Physics, Cutnell & Johnson, 6th Ed., Chapters 18, 21 (Wiley 2004)
Apparatus: Evacuated glass tube with accelerating electrode and electric deflecting plates, external magnetic deflecting current coils (Helmholtz configuration), power supplies, meters, Vernier Software's Graphical Analysis

CAUTION!!! THIS EXPERIMENT USES A VERY HIGH VOLTAGE POWER SUPPLY! BE EXTREMELY CAREFUL! DO NOT MAKE CONNECTIONS WITH THE POWER SUPPLY ON. DO NOT TOUCH THE HV WIRE OR ITS CONNECTORS WHEN THE SUPPLY IS ON. DON'T ALTER FILAMENTS.

WHEN IN DOUBT - ASK YOUR INSTRUCTOR

FIGURE 1: The fluorescent screen is gridded. The electron beam path produces fluorescence. You will read (x,y) coordinates along the path of a deflected beam (x horizontal, y vertical) and enter into a Graphical Analysis file, which will apply the path theory appropriate to the physical case and calculate quantities of interest. (In the simulations below, motion is in the (x,z) plane, whereas in our experimental axes, it is in the (x,y) plane.)
There are three possible arrangements: 

**Part A: Electric Deflection Field Only**  
As the electrons in the deflection region move in the horizontal (x) direction at constant speed, after acceleration to constant x-speed by potential $V_a$, an electric force may be applied in the vertical (y) direction by applying a deflecting potential difference $V_d$ between two horizontal deflecting plates, producing a changing $v_y$ (constant, non-zero $E_y$), with constant $v_x$. The electron "falls" upward under electric force, gaining speed, as a horizontally fired projectile falls downward under gravitational force. As for a projectile moving under constant gravitational acceleration, *the path is parabolic*.

The path does not depend on things such as the electron charge to mass ratio $\frac{e}{m}$, the deflecting voltage $V_d$, and the accelerating voltage $V_a$, but only involves the electric plate separation $d$:

$$y = \frac{x^2}{4d}$$  
Electric deflection path theory

so the only information fitting the deflection parabola gives is the value of $d$ (which can be obtained more easily and reliably by looking at the apparatus!)

This surprising result comes from the electrical system, in which there is only a single electrical power supply for both deflection and accelerating voltages: $V_d = V_a$.

See below for a detailed derivation.

**Part B: Magnetic Deflection Field Only**  
In addition, the apparatus permits deflection in the vertical plane (both x and y directions) by a magnetic force, produced by current in an external "Helmholtz coil" arrangement. A constant, external magnetic field acts at right angles to the motion, producing acceleration without work, so the speed remains constant. *The path is circular*.

As with a mass whirled on a string with constant tension, the radial magnetic force on the moving electron charge = electron mass x radial acceleration: $evB = \frac{mv^2}{R}$, giving

$$R = \frac{mv}{eB}$$  
where, assuming the circle point is (0,0), and any path (x,y) point gives R:

$$R = (\frac{x^2 + y^2}{2y})$$. (x,y) data is taken at several points for averaging.
See below for a detailed derivation.

**Part C: Crossed Electric and Magnetic Deflection Fields.** And, electric and magnetic deflections may be applied simultaneously. In general, the $\vec{E}$ and $\vec{B}$ forces are unbalanced, and the path is curved. If there is balance between $\vec{E}$ and $\vec{B}$ forces, the *Wien velocity filter path is a straight line*: $F_E = -F_B$ or $e|\vec{E}| = ev|\vec{B}|$, with the correct relative directions of $\vec{E}$ and $\vec{B}$.

See below for details.

**Appendix 1  Simulations**

![Charged particle motion in Electric / Magnetic Field](image)

**Figure 2  Simulation, Case A.** Electric field only.

Quadratic beam path under constant acceleration. The particle deflects opposite to the E field direction, because it is negatively charged. The (x,y) path is parabolic (quadratic fit). In our apparatus, the acceleration and deflections voltages cannot be varied independently.
Figure 3  Simulation, Case B  Constant external magnetic field only.

The particle moves in a circle perpendicular to the $\mathbf{B}$ field, at constant speed but non-constant velocity. The B field accelerates the particle (changes its direction) but does no work on it, because the velocity is always perpendicular to the field. Note negative charge. The path is circular.

In the actual experiment, only (x,y) points over a small part of the circular path will be observed.
Figure 4a  Simulation, Case C (Wien velocity filter)

Magnetic and electric forces are balanced, producing no deflection for the particular particle velocity. Check $\frac{|E|}{|B|}$ ratio and compare with velocity. Note negative charge. The path is linear.
Figure 4b  Simulation: Case C  ExB fields. B field too large to produce straight path, for the particle's velocity.

Introduction: The experiment employs a vacuum tube which allows unimpeded motion of electrons under the influence of applied electromagnetic fields.

The tube incorporates an "electron gun". An oxidized metal filament is heated (note glow in tube neck at right) so that some of the electrons of the metal get enough energy to leave the filament. They are then accelerated by an accelerating potential difference $V_a$, which gives them kinetic energy $\frac{1}{2} m v^2$, equal to $eV_a$. ($e$ is charge of an electron, $-1.6 \times 10^{-19}$ coulombs, and $m$ is its mass, $9.1 \times 10^{-31}$ kg.) The electron beam is made visible when the electrons strike a fluorescent screen.

A voltage of one to three thousand volts is applied across two horizontal conducting plates which are insulated from each other.

The upper plate is $a=$, the lower – (ground).

Current coils in series provide a horizontal magnetic field in the deflection region.

Figure 5 Evacuated tube for electron acceleration and evacuation, with electrical connections and external Helmholtz magnetic field current coils. Not shown is the internal hot filament electron source, which produces the glow seen in operation.

Appendix 2 Detailed procedures

Copy the supplied Graphical Analysis file and save under another name.

Enter data in each part as directed. Observe the graphical presentation of results and the ratios which give the relation between your experimental results, as interpreted in the simplifying geometric approximations to the actual physical system, and the results of assuming the simplifying results are correct. These ratios are typically around 1.
A. ELECTRIC DEFLECTION ONLY

Figure 6   Part A quadratic fit to electric field beam deflection.
Figure 7  Part A (Electric deflection) - top deflection plate is connected to the positive terminal of the large high voltage power supply (white cable), which also provides the acceleration voltage \( V_a \) \( (V_a = V_d) \). The bottom deflection plate is connected to the negative terminal (ground) of the large power supply (black cable).

Turn all voltages to zero and turn off the power supply. Turn off the magnet power supply.

With the high voltage power supply turned down and off, connect the + high voltage cable (white) from the high voltage power supply to the top deflection plate electrode feed through. Connect the ground (-) cable (black) from the power supply to the bottom plate. Note again that the same high voltage is applied both to the electron gun and to the deflection plates. Do not turn the power supply on until the instructor has checked your connections. Check that both the high voltage and the coil current control knobs are turned fully counter-clockwise. Turn on the power supply and observe the beam trace while varying the high voltage. At Busch, the small multimeter should be set to a scale of 20 V and its reading (in volts) should be multiplied by 500. At Douglass, use the 200 mV scale and multiply the meter reading in millivolts by 100 to get the high voltage output. (At Busch, a meter reading of 4 V \( \to \) 2,000 volts; at Douglass, a meter reading of 20 mV \( \to \) 2,000 volts.)

Verify that the beam trace locus does not depend on accelerating voltage. Set \( V_a \) to the lowest voltage that gives a clear and stable trace and leave it fixed. Enter into GA the
the values of $y_{beam}$ at $x_{beam} = 5, 6, \ldots$ cm. Remember to convert everything to SI units (specifically meters); mixing units will give you an erroneous result.

Why does the beam trace intensity increase with increasing high voltage?

**B. MAGNETIC FIELD DEFLECTION ONLY**

Part B (Magnetic deflection) See Figure 7.

Turn high voltage down and off. Disconnect the black ground wire!!!

Connect bottom deflection plate to top deflection plate via jumper cable (short cable with alligator clips at both ends).

**Helmholtz Coils**

A useful laboratory technique for getting a fairly uniform magnetic field is to use a pair of circular coils on a common axis with equal currents flowing in the same sense. For a given coil radius, you can calculate the separation needed to give the most uniform central field. This separation is equal to the radius of the coils. The magnetic field lines for this geometry are illustrated below.
The magnetic field on the centerline of a current loop can be calculated from the Biot-Savart law. The magnetic field from the two loops of the Helmholtz coil arrangement can be obtained by superimposing the two constituent fields.

Recheck that the high voltage and coil current supply control knobs are fully off (counter-clockwise) and that both supplies are switched off. Leave the high voltage cable connected to the upper deflection electrode but disconnect any power supply ground cable from the bottom electrostatic deflection plate. Next, connect the alligator clip lead (jumper cable) between the bottom and top plates, ensuring that charging of the plates from beam scatter does not develop a vertical electric field.

Ask your instructor to check the connections before turning the supply on. Only then turn the supply on.

Increase the high voltage until you first see a blue fluorescent beam trace on the screen. Turn on the magnet power supply and observe the deflection caused by the magnetic field, changing the magnetic field by varying the coil current. Observe the effect of turning down and reversing the current.

Increase the acceleration voltage to a value that gives a single, clear trace. For \( x \approx 0.06 \, \text{m} \), measure and input into GA the \( y \) deflection for four well spaced coil currents within your working range.

Switch off the high voltage, then turn off the magnet power supply. Wait for discharge. Reverse the direction of current through the coils by interchanging leads at the supply. Switch on the high voltage and repeat for four reverse polarity currents. The beam will be deflected in the opposite direction because the magnetic field is now in the opposite direction. Input the \( y \) deflections. GA will calculate bending radii \( R \), velocity values, and experimental \( \frac{e}{m} \) exp values. (The two different values of \( y \) for same acceleration voltage and opposite B fields will tend to cancel the effects of the earth's magnetic field and any misalignment of the screen.)
C. MAGNETIC AND ELECTRIC DEFLECTION

Part C  See Figure 7.

Turn high voltage down and switch off. Connections are the same as Part A, but magnet power supply is turned on.

Turn all power supplies down and off. Reconnect the electrical connections as in part A (removing shorting jumper between upper and lower electrostatic deflection plates, and attaching ground wire from power supply).

You will have to figure out by trial and error the sense of current $I_C$ to get this velocity filter to work (opposite B and E deflections.) Change the magnet current polarity if necessary; maintain the upper deflection plate at + high voltage. Connect the supply wires for the magnetic field-generating Helmholtz coils so that the deflection produced by the magnetic field will oppose that caused by the electric field. (The electric field is directed from + to - potential, by definition, but the electron charge is negative !) Turn on the magnet power supply. For several high voltage values $V_a (= V_d)$ vary the magnet coil current $I$ until the beam is horizontal at the central axis of the coil. Enter the balance values of $V_a$ and $I$ into GA, Part C. GA will calculate the Wien velocity

$$ v_{Wien} = \frac{E}{B} = \frac{V_a}{d} \frac{1}{4.23 \times 10^{-31}}, \quad v_a \text{ as } \sqrt{2 \frac{e}{m} V_a} \text{ using the accepted value of } \frac{e}{m} = 1.76 \times 10^{-11}. $$

Appendix 3  Detailed derivations

**Part A  Electric Deflection Field Only**  

Simplified model A:  Constant vertical $\vec{E}$ field with sharp boundary at $x = 0$. Electrons enter electric field region with full acceleration potential energy.

We will approximate the finite parallel conducting plate geometry using the infinite plate result that $\vec{E} = \frac{V_a}{d}$ is constant and perpendicular to the plates. Then there is no horizontal $x$ acceleration of a charged particle, and constant vertical $y$ acceleration (upward toward + plate for – charge).
Electric and Magnetic Forces

Deflection Force Equation 1a:

\[ \vec{F} = q \vec{E} = ma_{\text{theoretical}} \quad \Rightarrow \quad a_{\text{th}} = \text{constant, in y direction} \]

Then

\[ y = \frac{1}{2} a_{\text{th}} t^2 \quad \text{for horizontal entry to field region, with} \quad a_{\text{th}} = \frac{F}{m} = \frac{q E}{m} = \frac{e V_d}{m} \]

\[ y = \frac{1}{2} \frac{e}{m} \frac{V_d}{d} t^2 \]

and

\[ t^2 = \frac{x^2}{v_x^2} = \frac{m x^2}{2eV_a} \]

from the Acceleration Energy Conservation: Equation 1b:

\[ \frac{1}{2} m v_x^2 = eV_a \]

So, the electron path is quadratic in time, and also in x because x is proportional to time t:

\[ y = \frac{1}{2} \frac{e}{m} \frac{V_d}{d} \frac{mx^2}{2eV_a} = \frac{1}{2} \frac{1}{2d} \frac{V_d}{V_a} = \frac{x^2}{4d} \]

because the circuits are wired so that \( V_d = V_a \).

Note that, for this reason, the path is independent of accelerating voltage \( V_a \).

Why does not this expression contain the electron charge or mass? Because the horizontal acceleration voltage \( V_A \) acts on the same \( \frac{e}{m} \) as does the vertical acceleration voltage \( V_d \). Why do the voltages \( V_a \) and \( V_d \) not appear? Because a single high voltage power supply is used for both, for simplicity and economy.

Otherwise the ratio \( \frac{V_d}{V_a} \) would appear and the path would generally be different than ours. But, as long as the voltage ratio did not change, neither would the trace.

Turn up \( V_a (= V_d) \) until you see a clear blue beam trace. Read several pair values \((x, y)\) on the trace (in meters), and enter into Graphical Analysis Part A. Graph \( y \) vs. \( x \); Analyze: Curve Fit: Quadratic.

(GA’s quadratic fit function has the form \( a_{GA} x^2 + bx + c \) where \( a_{GA} \) is just a generic coefficient of \( x^2 \). Compare \( a_{GA} \) by ratio with \( \frac{1}{4d} : \frac{1}{4d} / a_{GA} \).

The deflection plates are not infinite and, although they are mechanically placed symmetrically placed above and below the system axis, they are not electrically symmetric.
Electric and Magnetic Forces

– the upper plate is at acceleration potential $V_a$, the same as the acceleration and beam defining structure, and the lower plate is at ground potential 0 (power supply reference) voltage.

**Part B Magnetic Deflection Field Only**

Simplified Model B: Constant horizontal $\vec{B}$ field with sharp cutoff at $x = 0$. Electrons enter magnetic field region with full acceleration potential energy.

A static magnetic field exerts a velocity dependent force on a moving charge at right angles to the motion, so it changes the direction of motion (i.e., the velocity direction) without doing any work on the particle (so constant velocity magnitude), so circular motion results:

**Magnetic Force: Equation 2**

$$|\vec{F}| = eB = \frac{\nu^2}{R} \quad \Rightarrow R = \frac{mv}{eB}$$

We get velocity $\nu$ from acceleration **Equation 2**

$$\frac{1}{2}mv^2 = eV_a \quad \Rightarrow \nu = \sqrt{\frac{2eV_a}{m}}$$

and $|\vec{B}|$ from $|\vec{B}| = 4.23 \times 10^{-3} I$ where $I$ is the common current through the two coils in series.

If we could see an entire circular path, we could determine $R$ and thus an experimental value of $\frac{e}{m}_{\text{exp}}$ to compare with the accepted value. But we can't. However, a partial arc of the circle should do. In fact, two points on the circle should do, along with the assumption that the circle's center lies directly above the $x = 0$ entrance point. (Remember, we assumed sharp field cut-offs at that point, for simplicity. See the figure in Appendix 2.) We will follow this approach (i.e., two points of the circle), finding different bending radii $R$ for a few different $|\vec{B}|$ values (different currents $I$). We will also reverse coil current direction to partially cancel effects of coil misalignment and of (small) earth magnetic field components.
Figure 9  Geometry of electron deflection in a constant magnetic field region having a sharp boundary at x = 0 (beam horizontal entry point). Because of the (assumed) sharp boundary, the initially horizontal beam starts to bend at x = 0; therefore, the center of the circular path lies above the entry point, i.e. at x = 0.

The electron beam travels to the left and becomes visible on the fluorescent screen. The circular motion is caused by the magnetic field directed into the page. (If the beam bent down, the field would be directed out of the paper. Remember that the electron charge is negative!)

The diagram shows that, for the deflected beam path, any x-y pair on the beam path gives the (B-dependent) bending radius R from

\[ R^2 = (R-y)^2 + x^2 = (R^2 - 2Ry + y^2) + x^2 \rightarrow R = \frac{x^2 + y^2}{2y} . \]

If we could measure x and y (points on the circle) from the center, x and y would be related by \( x^2 + y^2 = R^2 \) But our x, y origin (0,0) lies at the bottom of the circle, so the relation is (after a little geometry)

\[ R = \frac{x^2 + y^2}{2y} . \]

(0,0) will be one of our two points for each circle, and observed (x,y) values along the circle will be the others. (These should give the same R, in principle.) Then

\[ \frac{e}{m} = \frac{2V_a}{(B^2 R^2)} \quad \text{where} \quad R^2 = \left( \frac{x^2 + y^2}{2y} \right)^2 \quad \text{and} \quad |B| = 4.23 \times 10^{-3} I . \]
Set and record fixed $V_a$ at fixed $I_B$; then observe and enter $(x, y)$ and $I_B$ for four well-spaced values of $x$ over the circular beam path. Substitute your value of the acceleration voltage $V_a$ into the definition of $\frac{e}{m} \text{exp}$. Reverse the current direction and repeat ($y$ signs should reverse). (The + and – current values should have approximately equal magnitudes, to average asymmetries and earth field effects.)

GA will calculate R's for each entry, but you must insert your own operating value of accelerating voltage $V_a$ into the definition of calculated column $\frac{e}{m} \text{exp}$.

GA will also plot the ratio $\frac{e}{m} \text{exp}$ vs. observation number $N$. Examine for gross discrepancies. Analyze: Statistics. Record the mean value of $\frac{e}{m} \text{exp}$ and the standard deviation.

GA will calculate and plot the ratio $\frac{e}{m} \text{exp} / \frac{e}{m} \text{accepted} = \frac{e}{m} \text{exp} / 1.76 \times 10^{11}$.

Observe for systematic trend. Analyze: Statistics. Is the mean ratio near to 1?

**Note:** There is no theory giving numerical values of $e$ and $m$; they are taken as fundamental experimental facts of our universe. “String” theory is trying.

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**Part C Crossed Electric and Magnetic Deflection Fields Simplified model C:**

Constant vertical $\vec{E}$ field and constant horizontal $\vec{B}$, each with sharp boundary at $x = 0$. Electrons enter electric field region with full acceleration potential energy.

The electric and magnetic fields are at right angles. But the magnetic field force produces force at right angles to its direction (and that of the velocity vector), so both fields act in the vertical direction, either aiding or opposing. We will seek the current direction so that the two forces oppose and so that the two force magnitudes cancel, producing zero deflection. From the electric and magnetic force equations...
Electric and Magnetic Forces

\[ q \vec{E} = qv \vec{B} \quad \text{or} \quad v = \frac{|E|}{|B|}, \]

a velocity condition independent of charge magnitude or sign. Such a condition is obtained (by adjustment of \( \vec{E}, \vec{B} \) or both) to select beam particles of a desired velocity (Wien velocity filter). Others will be deflected one way or the other, according as their velocities are greater or less than that selected for.

Set a few accelerating voltages \( V_a \), then adjust magnetic field \( B \) in each case by adjustment of coil current \( I \) to obtain zero deflection (this won't be exact - look for horizontal beam in center of deflection region). Then

\[ v_{\text{Wien}} = \frac{|E|}{|B|} = \frac{V_d}{d} / 4.23 \times 10^{-3} I = \frac{V_a}{d} / 4.23 \times 10^{-3} I \]

(remember \( V_d = V_a \)), which can be compared with the velocity predicted from the acceleration energy conservation Equation 2 above

\[ v_a = \sqrt{\frac{e}{m}} \frac{2V_a}{d}. \]

Enter into Graphical Analysis Part C several values of \( V_a \) and \( I \) that produce zero deflection, and let GA calculate the Wien velocity \( v_{\text{Wien}} \) and the comparison velocity \( v_a \), obtained from acceleration energy. In calculating \( v_a \), we will use the accepted value

\[ \frac{e}{m_{\text{accepted}}} = 1.76 \times 10^{11} \text{(mks units)}. \]

GA will also calculate and plot the ratio \( \frac{v_{\text{Wien}}}{v_a} \). Examine the plot for any systematic trend. Analyze: Statistics. Does the mean ratio differ substantially from 1?