Problem 7.6d

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As sigma_1 sets the scale for sigma we can choose it to be 1, and use tau for ct. gamma is a reserved word, so I use gam for it. Let xp and yp be the sigma derivatives of x and y. So

```maple
> xp := (tau, sigma) -> cos(gam * sin(Pi*tau)*sin(Pi*sigma)) * cos(gam * cos(Pi*tau)*cos(Pi*sigma));
xp := (τ, σ) → cos(gam sin(π τ) sin(π σ)) cos(gam cos(π τ) cos(π σ))

> yp := (tau, sigma) -> cos(gam * sin(Pi*tau)*sin(Pi*sigma)) * sin (gam * cos(Pi*tau)*cos(Pi*sigma));
yp := (τ, σ) → cos(gam sin(π τ) sin(π σ)) sin(gam cos(π τ) cos(π σ))
```

The sigma=0 endpoint is at x=y=0, so x and y are just the integral of xp and yp from 0 to sigma

```maple
> x := (tau, sigma) -> int(xp(tau, s), s=0..sigma);
x := (τ, σ) → \int_0^σ xp(τ, s) \, ds

> y := (tau, sigma) -> int(yp(tau, s), s=0..sigma);
y := (τ, σ) → \int_0^σ yp(τ, s) \, ds
```

The book asks us to use

```maple
> gam := Pi/sqrt(2);
gam := π/\sqrt{2}
```

First let’s look at t=0:

```maple
> plot([[x(0, sig), y(0, sig), sig=0..1]]);
> plot(y(0,sig), sig=0..1);

> plot(x(0,sig), sig=0..1);
Let's evaluate $a$ for this gamma, with the scale set by $\sigma_1=1$:
\[
> \text{evalf}(x(0,1));
\]
\[
0.09847494076
\]

$\sigma_1$ sets the scale, all that is really determined is $a/\sigma_1$. So if we want $a=1$, we need to set $\sigma_1$ to the inverse of this:
\[
> \frac{1}{%};
\]
\[
10.15486775
\]

which agrees with the book. Just to check on the bessel function $J_0(gam)$
\[
> \text{evalf}(BesselJ(0,gam));
\]
\[
0.09847494081
\]

Now let's look at the string at time $\sigma_1/4$:
\[
> \text{plot}([x(1/4,sig),y(1/4,sig),sig=0..1]);
\]
Was that a kink? Let's blow up the region, which looks like it would be at \( \sigma = 1/2 \):
\[
\text{plot}([x(1/4, \sigma), y(1/4, \sigma), \sigma = 0.49..0.51]);
\]

That looks kinky, but also shows numerical method imprecision on the screen. (For some reason, the printed version seems more accurate). Let's try again:
\[
\text{plot}([x(1/4, \sigma), y(1/4, \sigma), \sigma = 0.45..0.55]);
\]
I guess that’s a kink. No question here, though, at $t=\sigma_{1/3}$:

```maple
plot([x(1/3, sig), y(1/3, sig), sig=0..1]);
```