1. A certain O-type star has \( M = 30 M_\odot \), \( R = 6.6 R_\odot \), \( X = 0.70 \), and \( Y = 0.30 \). Estimate (a) the importance of radiation pressure and (b) the central temperature by approximating the star by the standard model. Compare your results to the more accurate values obtained by numerical calculation of \( \beta_c = 0.77 \) and \( T_c = 3.7 \times 10^7 \) K.

2. To a good approximation a white dwarf can be considered to be a star supported by the pressure of completely degenerate electrons. As the mass is increased, the central density becomes so high that the degeneracy becomes relativistic at the center, such that (confirm this and show your work)

\[
P_c \to 1.244 \times 10^{15} (\rho/\mu_e)^{4/3} \text{ dynes cm}^{-2}
\]

and falls off to nonrelativistic degeneracy in the outer portions of the star. As the mass is continually increased, the star shrinks to ever higher densities and ever smaller radii, until the electrons become highly relativistic everywhere. Then the equation of state given above becomes applicable throughout the star. Show that at this point the mass is

\[
M = \frac{5.80}{\mu_e^2} M_\odot.
\]

This is the Chandrasekhar mass limit for white dwarfs. Evaluate it for a pure carbon composition.

3. Show that \( MR^3 = \text{constant} \) for polytropes described by a nonrelativistic, completely degenerate equation of state. Adding mass to a white dwarf causes its radius to decrease.