1. Derive the Saha equation from the following elements:
   (a) the steady-state condition of photoionization/recombination:
   \[
   N_{r+1} n_e(p) [A_{r+1,r} + u(\nu_{r,r+1}) B_{r+1,r}] = N_r u(\nu_{r,r+1}) B_{r,r+1}
   \]
   (b) the ratio of the Einstein B coefficients:
   \[
   \frac{B_{r,r+1}}{B_{r+1,r}} = \frac{g_{r+1} g_e 4\pi p^2}{g_r h^3}
   \]
   (c) a Maxwell-Boltzmann distribution for \( n_e(p) \)
   (d) and the photon energy density \( u(\nu) \) in thermal equilibrium.

2. Following a procedure similar to what was done in class for hydrogen, find the temperature at the middle of the He II partial ionization zone, where half of the He II atoms have been ionized, for a stellar atmosphere composed of pure helium. (A certain class of white dwarfs has this type of atmosphere.) The ionization energies of neutral and singly ionized helium are \( \chi_I = 24.6 \) eV and \( \chi_{II} = 54.5 \) eV, respectively. The partition functions are \( Z_I = 1 \), \( Z_{II} = 2 \), and \( Z_{III} = 1 \). This ionization zone is found at a greater depth in the star than either the He I or H partial ionization zones, so the electron pressure is larger—use a value of \( P_e = 10^4 \) dyne cm\(^{-2} \).
   (a) Find \( N_{II}/N_I \) and \( N_{III}/N_{II} \) for temperatures of 10,000 K, 35,000 K, and 60,000 K.
   (b) Express \( N_{III}/N_{total} \) in terms of the ratios \( N_{II}/N_I \) and \( N_{III}/N_{II} \).
   (c) Make a graph of \( N_{III}/N_{total} \) for a range of temperatures from 10,000 K to 60,000 K. What is the temperature at the middle of the He II partial ionization zone?