Week 10 homework

IMPORTANT NOTE ABOUT WEBASSIGN:
In the WebAssign versions of these problems, various details have been changed, so that the answers will come out differently. The method to find the solution is the same, but you will need to repeat part of the calculation to find out what your answer should have been.

WebAssign Problem 1: When the temperature of a coin is raised by 75 °C, the coin’s diameter increases by $2.3 \times 10^{-5}$ m. If the original diameter of the coin is $1.8 \times 10^{-2}$ m, find the coefficient of linear expansion.

**REASONING AND SOLUTION** The change in the coin’s diameter is $\Delta d = \alpha d_0 \Delta T$, according to Equation 12.2. Solving for $\alpha$ gives

$$\alpha = \frac{\Delta d}{d_0 \Delta T} = \frac{2.3 \times 10^{-5} \text{ m}}{(1.8 \times 10^{-2} \text{ m})(75 \degree \text{C})} = 1.7 \times 10^{-5} (\degree \text{C})^{-1} \quad (12.2)$$

WebAssign Problem 2: Interactive Solution 12.47 at www.wiley.com/college/cutnell deals with one approach to solving problems such as this. A 0.35-kg coffee mug is made from a material that has a specific heat capacity of 920 J/(kg·C°) and contains 0.25 kg of water. The cup and water are at 15 °C. To make a cup of coffee, a small electric heater is immersed in the water and brings it to a boil in three minutes. Assume that the cup and water always have the same temperature and determine the minimum power rating of this heater.

**REASONING** According to Equation 6.10b, the average power is the change in energy divided by the time. The change in energy in this problem is the heat supplied to the water and the coffee mug to raise their temperature from 15 to 100 °C, which is the boiling point of water. The time is given as three minutes (180 s). The heat $Q$ that must be added to raise the temperature of a substance of mass $m$ by an amount $\Delta T$ is given by Equation 12.4 as $Q = cm\Delta T$, where $c$ is the specific heat capacity. This equation will be used for the water and the material of which the mug is made.

**SOLUTION** Using Equation 6.10b, we write the average power $\bar{P}$ as

$$\bar{P} = \frac{\text{Change in energy}}{\text{Time}} = \frac{Q_{\text{Water}} + Q_{\text{Mug}}}{\text{Time}}$$

The heats $Q_{\text{Water}}$ and $Q_{\text{Mug}}$ each can be expressed with the aid of Equation 12.4, so that we obtain
\[
\overline{P} = \frac{Q_{\text{Water}} + Q_{\text{Mug}}}{\text{Time}} = \frac{c_{\text{Water}} m_{\text{Water}} \Delta T + c_{\text{Mug}} m_{\text{Mug}} \Delta T}{\text{Time}}
\]

\[
= \frac{[4186 \text{ J/(kg} \cdot \text{°C})][0.25 \text{ kg}][100.0 \text{ °C} - 15 \text{ °C}]}{180 \text{ s}}
+ \frac{[920 \text{ J/(kg} \cdot \text{°C})][0.35 \text{ kg}][100.0 \text{ °C} - 15 \text{ °C}]}{180 \text{ s}} = 650 \text{ W}
\]

The specific heat of water has been taken from Table 12.2.

**WebAssign Problem 3:** A person eats a container of strawberry yogurt. The Nutritional Facts label states that it contains 240 Calories (1 Calorie = 4186 J). What mass of perspiration would one have to lose to get rid of this energy? At body temperature, the latent heat of vaporization of water is \(2.42 \times 10^6\) J/kg.

**REASONING** As the body perspires, heat \(Q\) must be added to change the water from the liquid to the gaseous state. The amount of heat depends on the mass \(m\) of the water and the latent heat of vaporization \(L_v\), according to \(Q = mL_v\) (Equation 12.5).

**SOLUTION** The mass of water lost to perspiration is

\[
m = \frac{Q}{L_v} = \frac{(240 \text{ Calories}) \left( \frac{4186 \text{ J}}{1 \text{ Calorie}} \right)}{2.42 \times 10^6 \text{ J/kg}} = 0.42 \text{ kg}
\]

**WebAssign Problem 4:** A mass of 135 g of a certain element is known to contain \(30.1 \times 10^{23}\) atoms. What is the element?

**REASONING AND SOLUTION** The number \(n\) of moles contained in a sample is equal to the number \(N\) of atoms in the sample divided by the number \(N_A\) of atoms per mole (Avogadro’s number):

\[
n = \frac{N}{N_A} = \frac{30.1 \times 10^{23}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 5.00 \text{ mol}
\]

Since the sample has a mass of 135 g, the mass per mole is

\[
\frac{135 \text{ g}}{5.00 \text{ mol}} = 27.0 \text{ g/mol}
\]

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic mass of the substance. Therefore, the atomic mass is 27.0 u. The periodic table of the elements reveals that the unknown element is aluminum.
**WebAssign Problem 5:** What is the density (in kg/m$^3$) of nitrogen gas (molecular mass = 28 u) at a pressure of 2.0 atmospheres and a temperature of 310 K?

**REASONING** According to Equation 11.1, the mass density $\rho$ of a substance is defined as its mass $m$ divided by its volume $V$: $\rho = m/V$. The mass of nitrogen is equal to the number $n$ of moles of nitrogen times its mass per mole: $m = n$ (Mass per mole). The number of moles can be obtained from the ideal gas law (see Equation 14.1) as $n = (PV)/(RT)$. The mass per mole (in g/mol) of nitrogen has the same numerical value as its molecular mass (which we know).

**SOLUTION** Substituting $m = n$ (Mass per mole) into $\rho = m/V$, we obtain

$$\rho = \frac{m}{V} = \frac{n \text{(Mass per mole)}}{V}$$

(1)

Substituting $n = (PV)/(RT)$ from the ideal gas law into Equation 1 gives the following result:

$$\rho = \frac{n \text{(Mass per mole)}}{V} = \frac{P \text{(Mass per mole)}}{RT}$$

The pressure is 2.0 atmospheres, or $P = 2 \times 1.013 \times 10^5$ Pa. The molecular mass of nitrogen is given as 28 u, which means that its mass per mole is 28 g/mol. Expressed in terms of kilograms per mol, the mass per mole is

$$\text{Mass per mole} = \left(\frac{28 \text{ g}}{\text{mol}}\right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right)$$

The density of the nitrogen gas is

$$\rho = \frac{P \text{(Mass per mole)}}{RT} = \frac{2 \times 1.013 \times 10^5 \text{ Pa}}{(8.31 \text{ J/(mol·K)}}) \left(\frac{28 \text{ g}}{\text{mol}}\right) \left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right) \left(\frac{310 \text{ K}}{1 \text{ K}}\right) = 2.2 \text{ kg/m}^3$$

**WebAssign Problem 6:** A gas fills the right portion of a horizontal cylinder whose radius is 5.00 cm. The initial pressure of the gas is $1.01 \times 10^5$ Pa. A frictionless movable piston separates the gas from the left portion of the cylinder, which is evacuated and contains an ideal spring, as the drawing shows. The piston is initially held in place by a pin. The spring is initially unstrained, and the length of the gas-filled portion is 20.0 cm. When the pin is removed and the gas is allowed to expand, the length of the gas-filled
chamber doubles. The initial and final temperatures are equal. Determine the spring constant of the spring.

**REASONING AND SOLUTION** The volume of the cylinder is \( V = AL \) where \( A \) is the cross-sectional area of the piston and \( L \) is the length. We know \( P_1 V_1 = P_2 V_2 \) so that the new pressure \( P_2 \) can be found. We have

\[
P_2 = \frac{P_1 V_1}{V_2} = P_1 \left( \frac{A L_1}{A_2 L_2} \right) = P_1 \left( \frac{L_1}{L_2} \right) \quad \text{(since } A_1 = A_2) \]

\[
= \left( 1.01 \times 10^5 \text{ Pa} \right) \left( \frac{L}{2L} \right) = 5.05 \times 10^4 \text{ Pa}
\]

The force on the piston and spring is, therefore,

\[
F = P_2 A = (5.05 \times 10^4 \text{ Pa}) \pi (0.0500 \text{ m})^2 = 397 \text{ N}
\]

The spring constant is \( k = F/x \) (Equation 10.1), so

\[
k = \frac{F}{x} = \frac{397 \text{ N}}{0.200 \text{ m}} = 1.98 \times 10^3 \text{ N/m}
\]

**WebAssign Problem 7:** Refer to Multiple-Concept Example 6 for insight into the concepts used in this problem. An oxygen molecule is moving near the earth’s surface. Another oxygen molecule is moving in the ionosphere (the uppermost part of the earth’s atmosphere) where the Kelvin temperature is three times greater. Determine the ratio of the translational rms speed in the ionosphere to that near the earth’s surface.

**REASONING** The translational rms-speed \( v_{\text{rms}} \) is related to the Kelvin temperature \( T \) by

\[
\frac{1}{2} m v_{\text{rms}}^2 = \frac{3}{2} kT \quad \text{(Equation 14.6), where } m \text{ is the mass of the oxygen molecule and } k \text{ is Boltzmann’s constant. Solving this equation for the rms-speed gives }
\]

\[
v_{\text{rms}} = \sqrt{\frac{3kT}{m}}. \text{ This relation will be used to find the ratio of the speeds.}
\]

**SOLUTION** The rms-speeds in the ionosphere and near the earth’s surface are
\[ (v_{\text{rms}})_{\text{ionosphere}} = \sqrt{\frac{3kT_{\text{ionosphere}}}{m}} \quad \text{and} \quad (v_{\text{rms}})_{\text{earth's surface}} = \sqrt{\frac{3kT_{\text{earth's surface}}}{m}} \]

Dividing the first equation by the second gives

\[
\frac{(v_{\text{rms}})_{\text{ionosphere}}}{(v_{\text{rms}})_{\text{earth's surface}}} = \sqrt{\frac{3kT_{\text{ionosphere}}}{3kT_{\text{earth's surface}}}} = \sqrt{\frac{T_{\text{ionosphere}}}{T_{\text{earth's surface}}}} = \frac{\sqrt{3}}{1} = 1.73
\]

**WebAssign Problem 8:** At the start of a trip, a driver adjusts the absolute pressure in her tires to be $2.81 \times 10^5$ Pa when the outdoor temperature is 284 K. At the end of the trip she measures the pressure to be $3.01 \times 10^5$ Pa. Ignoring the expansion of the tires, find the air temperature inside the tires at the end of the trip.

**REASONING AND SOLUTION** To find the temperature $T_2$, use the ideal gas law with $n$ and $V$ constant. Thus, $P_1/T_1 = P_2/T_2$. Then,

\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right) = (284 \text{ K}) \left( \frac{3.01 \times 10^5 \text{ Pa}}{2.81 \times 10^5 \text{ Pa}} \right) = 304 \text{ K} \]

**WebAssign Problem 9:** A spherical balloon is made from a material whose mass is 3.00 kg. The thickness of the material is negligible compared to the 1.50-m radius of the balloon. The balloon is filled with helium (He) at a temperature of 305 K and just floats in air, neither rising nor falling. The density of the surrounding air is 1.19 kg/m$^3$. Find the absolute pressure of the helium gas.

**REASONING AND SOLUTION** We need to determine the amount of He inside the balloon. Begin by using Archimedes’ principle; the balloon is being buoyed up by a force equal to the weight of the air displaced. The buoyant force, $F_b$, therefore, is equal to

\[ F_b = mg = \rho Vg = \left[ 1.19 \text{ kg/m}^3 \right] \left[ \frac{4}{3} \pi \left( 1.50 \text{ m} \right)^3 \right] \left( 9.80 \text{ m/s}^2 \right) = 164.9 \text{ N} \]

Since the balloon has a mass of 3.00 kg (weight = 29.4 N), the He inside the balloon weighs $164.9 \text{ N} – 29.4 \text{ N} = 135.5 \text{ N}$. Hence, the mass of the helium present in the balloon is $m = 13.8 \text{ kg}$. Now we can determine the number of moles of He present in the balloon:

\[ n = \frac{m}{M} = \frac{13.8 \text{ kg}}{4.0026 \times 10^{-3} \text{ kg/mol}} = 3450 \text{ mol} \]
Using the ideal gas law to find the pressure, we have

\[ P = \frac{nRT}{V} = \frac{(3450 \text{ mol})[8.31 \text{ J/(mol K)}](305 \text{ K})}{\frac{4}{3} \pi (1.50 \text{ m})^3} = 6.19 \times 10^5 \text{ Pa} \]
Practice conceptual problems:

Chapter 12:

6. A simple pendulum is made using a long thin metal wire. When the temperature drops, does the period of the pendulum increase, decrease, or remain the same? Account for your answer.

**REASONING AND SOLUTION**  A simple pendulum is made using a long thin metal wire. From Equations 10.6 and 10.16, we know that the period of the pendulum is proportional to \( L/\sqrt{g} \), where \( L \) is the length of the wire. When the temperature drops, the length of the wire decreases; therefore, the period of the pendulum decreases.

10. Suppose liquid mercury and glass both had the same coefficient of volume expansion. Explain why such a mercury-in-glass thermometer would not work.

**REASONING AND SOLUTION**  An ordinary mercury-in-glass thermometer works on the following principle. When the thermometer is placed in contact with the object whose temperature is to be measured, the thermometer comes into thermal equilibrium with the object. As thermal equilibrium is reached, the glass tube and the enclosed mercury expand or contract, depending on whether the thermometer heats up or cools down. The coefficient of volume expansion of mercury is about 7 times greater than that of ordinary glass. Thus, the expansion or contraction of the mercury column relative to the scale on the glass is evident when one reads the thermometer. If the coefficients of volume expansion of mercury and glass were both the same, both would expand or contract by the same amount. The reading on the thermometer would never change.

13. Two different objects are supplied with equal amounts of heat. Give the reason(s) why their temperature changes would not necessarily be the same.

**REASONING AND SOLUTION**  When an amount of heat \( Q \) is added to an object of mass \( m \), its temperature will increase. According to Equation 12.4, the amount by which the temperature will increase is given by \( \Delta T = Q/cm \), where \( c \) is the specific heat capacity of the object.

Two different objects are supplied with equal amounts of heat. The objects could have the same specific heat capacities, but they could have different masses. The objects could have the same masses, but they could have different specific heat capacities. Or, the objects could have both different masses and different specific heat capacities. In any of these cases, the temperature changes of the objects would not be the same.

23. Medical instruments are sterilized under the hottest possible temperatures. Explain why they are sterilized in an autoclave, which is a device that is essentially a pressure cooker and heats the instruments in water under a pressure greater than one atmosphere.
**REASONING AND SOLUTION**  If medical instruments were sterilized in an open container of water, the water would be subjected to atmospheric pressure. Therefore, the highest possible temperature that could be attained is 100 °C. In order to raise the temperature above 100 °C, we would have to raise the pressure above the liquid, as suggested by Figure 12.33. An autoclave heats medical instruments in water under high pressure. We can see from Figure 12.33 that at pressures above atmospheric pressure, the boiling point of water is greater than 100 °C. Therefore, by sterilizing medical instruments in an autoclave, it is possible to attain temperatures much greater than 100 °C.

*Chapter 14:*

1. (a) Which, if either, contains a greater number of molecules, a mole of hydrogen \((H_2)\) or a mole of oxygen \((O_2)\)? (b) Which one has more mass? Give reasons for your answers.

**REASONING AND SOLUTION**

a. Avogadro's number \(N_A\) is the number of particles per mole of substance. Therefore, one mole of hydrogen gas \((H_2)\) and one mole of oxygen gas \((O_2)\) contain the same number (Avogadro's number) of molecules.

b. One mole of a substance has a mass in grams that is equal to the atomic or molecular mass of the substance. The molecular mass of oxygen is greater than the molecular mass of hydrogen. Therefore, one mole of oxygen has more mass than one mole of hydrogen.

8. A slippery cork is being pressed into a very full (but not 100% full) bottle of wine. When released, the cork slowly slides back out. However, if some wine is removed from the bottle before the cork is inserted, the cork does not slide out. Account for these observations in terms of the ideal gas law.

**REASONING AND SOLUTION**  A slippery cork is being pressed into a very full bottle of wine. When released, the cork slowly slides back out. If some of the wine is removed from the bottle before the cork is inserted, the cork does not slide out. When the bottle is very full, the volume of air in the bottle above the wine is relatively small. Therefore, pushing the cork in reduces the volume of that air by an appreciable fraction. As a result, the pressure of the air increases appreciably and becomes large enough to push the slippery cork back out of the bottle. If some of the wine is removed, the volume of air above the wine is much larger to begin with, and pushing the cork in reduces the volume of that air by a much smaller fraction. Consequently, the pressure of the air increases by a much smaller amount and does not become large enough to push the cork back out.

13. It is possible for both the pressure and volume of a monatomic ideal gas to change without causing the internal energy of the gas to change. Explain how this could occur.
**REASONING AND SOLUTION** According to Equation 14.7, the internal energy of a sample consisting of \( n \) moles of a monatomic ideal gas at Kelvin temperature \( T \) is \( U = \frac{1}{2} nRT \); therefore, the internal energy of such an ideal gas depends only on the Kelvin temperature. If the pressure and volume of this sample is changed *isothermally*, the internal energy of the ideal gas will remain the same. Physically, this means that the experimenter would have to change the pressure and volume in such a way, that the product \( PV \) remains the same. This can be verified from the ideal gas law (Equation 14.1), \( PV = nRT \). If the values of \( P \) and \( V \) are varied so that the product \( PV \) remains constant, then \( T \) will remain constant and, from Equation 14.7, the internal energy of the gas remains the same.

16. In the lungs, oxygen in very small sacs called alveoli diffuses into the blood. The diffusion occurs directly through the walls of the sacs. The walls are very thin, so the oxygen diffuses over a distance \( L \) that is quite small. Because there are so many alveoli, the effective area \( A \) across which diffusion occurs is very large. Use this information, together with Fick’s law of diffusion, to explain why the mass of oxygen per second that diffuses into the blood is large.

**REASONING AND SOLUTION** Fick's law of diffusion relates the mass \( m \) of solute that diffuses in a time \( t \) through a solvent contained in a channel of length \( L \) and cross-sectional area \( A \): \( m = (DA\Delta C)t / L \), where \( \Delta C \) is the concentration difference between the ends of the channel and \( D \) is the diffusion constant.

In the lungs, oxygen in very small sacs (alveoli) diffuses into the blood. The walls of the alveoli are thin, so the oxygen diffuses over a small distance \( L \). Since the number of alveoli is large, the effective area \( A \) across which diffusion occurs is very large. From Fick's law, we see that the mass of oxygen that diffuses per unit time is directly proportional to the effective cross-sectional area \( A \) and inversely proportional to the diffusion distance \( L \). Since \( A \) is large and \( L \) is small, the mass of oxygen per second that diffuses into the blood is large.