Newton's Second Law (Part I and Part II) 11-MAC

Purpose:

- To gain experience creating a presentable graph of data points, making a linear fit to data, and understand the significance of the correlation coefficient.
- To demonstrate Newton’s Second Law on a level track
- Verify Newton’s Second Law prediction on an inclined track.
- To calculate the force of friction.

Apparatus

Pasco track and sonic ranger (shown below), Pasco cart, LabPro interface and cables, friction block, force sensor, cart-force sensor adaptor, motion sensor, pulley, small bucket & long thin string, thicker short string, assorted small weights (of 5, 10, 20 gram mass), large weights (of 500 gram mass metal bar, 4 for each table), 2000 gram scale.

Background

Newton's Laws of Motion
Isaac Newton formulated three fundamental laws of motion:

1. Every object continues in its state of rest or of uniform speed in a straight line unless it is compelled to change that state by a net force acting on it.

2. The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass. The direction of the acceleration is in the direction of the applied net force.

3. When one object exerts a force on a second object, the second object exerts an equal and opposite force on the first.

The First Law is not intuitively obvious from everyday experience. Friction, an “unseen” force, slows things down. Thus, Aristotelian philosophers taught that everything naturally tended to slow down and come to rest.

The Second Law is essentially the equation \( F = ma \). (Symbols in bold-face are vectors.) Note that \( F \) is the net force; there can be more than one force in many instances. Their vector sum is the net force.

The Third Law is that the force \( F_1 \) by ‘1’ on ‘2’ is accompanied by an opposite force \( F_2 \) by ‘2’ on ‘1’ such that: \( F_1 = -F_2 \) or \( F_1 + F_2 = 0 \). That is, you push on something and you feel an opposite force of equal magnitude.

Note that the First and Second Laws refer to a single object. The Third Law involves two interacting objects; the two forces always act on different objects.

**Graphing**

The process of fitting a plot of experimental data to a mathematical curve is called a regression analysis (or least-squares fit). If the curve used is a straight line, the analysis is called a linear fit. Of course, if there is an experimental error, the fit will not be perfect. The correlation coefficient \( R \) is a number between -1 and +1 that tells how well the data can be fit to a straight line. If \( R = +1 \), the fit is perfect and the line has a positive slope. If \( R = -1 \), the fit is also perfect but the line has a negative slope. If \( R = 0 \), the data are uncorrelated. To do a linear regression, select “Analyze → Linear Fit” in the “Logger Pro” program in the folder “Lab Apps” on your computer’s Desktop. The program will fit the data to a straight line and give you the equation for the best-fit line with intercept, slope and correlation \( R \).

If presented appropriately, a graph can present data in a concise manner that is easy to understand and clearly shows the correlations and significance. A well-done graph has the following features:

**Title:** The title should give an overall description of the experiment. For example, if you are plotting the speeds of a ball and a feather dropped from a height, the title could be “Speed of Various Falling Objects”. Your title should not just repeat the information already given by the axes labels; therefore “v versus t” or “Speed versus Time” are not acceptable titles.
Axes: Both axes should be clearly labeled with the name of the quantity being plotted and the units: Time (s) and Speed (m/s). The range of values plotted should be chosen so the data fill most of the graph. For example, if the feather takes 3 s to fall, the x-axis should be from 0 to 3 s or better yet 0 to 4 s, but definitely not 0 to 10 s.

Data points: The data points should be large enough to be easily seen and clearly differentiated from other data. For example, the feather data might be plotted with solid circles and the ball with open squares. Data points should not be connected with a line.

Legend: If more than one set of data is plotted on the same graph, the graph should have a legend: circle – feather, square – ball. If you have fit the data to an equation, the equation and the parameters should be given on the graph: \( v = 3.2 + 9.8 t \).

In summary, graphing is an important part of your scientific communication. You need to spend time making sure that the graph is clear and communicates the ideas you want to convey. You can modify the above features by putting the pointer in the graph area and right clicking to choose “Cartesian Graph Options.” There are many other options you can explore.

Activity 1:(20 min) Graphing.

In order to get experience using Logger Pro and to understand the significance of the correlation coefficient R, do the following exercises:

A. Open “Logger” in “Lab Apps” and fill the X column with consecutive integers from 1 to 20. In the Y column, type in the first 20 digits of the number \( \pi \approx 3.1415926535897932384 \). So you have \((X,Y) = (1,3), (2,1), (3,4), \ldots (20,4)\). A scatter plot of X (horizontal) versus Y (vertical axis) should appear in the graph area. Would you expect a low (close to 0) or a high value (close to 1) of correlation R if you make a simple (linear fit) to your data? Make a linear fit (by Analyze \( \rightarrow \) Linear Fit) and record the resulting correlation R.

B. If the values are such that \((X,Y) = (1,1), (2,2), (3,3) \ldots\), the points lie on the straight line \( Y = X \). Now increase each value of Y in this case by adding to it 0.1 times the digits of \( \pi \) in the range \( 1 \leq X \leq 20 \). The values then become \((1,1.3), (2,2.1), (3,3.4), \ldots\) Predict the correlation R for a linear fit. Record the result given by the Linear Fit function.

Activity 2:(1 hr 40 min) Newton's Second Law.

You will now carry out an experiment to determine how force, acceleration and mass are related. The idea of the experiment is to measure the acceleration of the cart of mass M (using a sonic ranger) as it is pulled along the horizontal low friction track by a string that passes over a pulley and is attached to a vertically falling mass m. With a second probe (force probe) you can measure the tension in the string, which is the net force on the cart.
PART II.

Activity 3: (1 hr) Fixed Force. In this experiment the fixed net force is the force of gravity on the fixed weight $m$ in the basket. The moving system is the small mass in the basket and the cart and the metal bars on top of the cart. Therefore the total mass of the system can be changed, while the gravitational force is fixed. As a result the measured acceleration will also change.

Activity 4: (1 hr) Predictions using Newton’s Second Law. Your experiment in Activity 2 and 3 has hopefully demonstrated a linear correlation between force and acceleration, and Newton’s second law says that it is the net force that produces the acceleration. To test whether Newton’s second law is applicable to other situations, you will now tilt the track up at an angle of about $7^\circ$ or less, so that the cart will have to move up-hill when pulled by the weight. You will:

i. Calculate the weight needed to just hold the cart in static equilibrium and then compare your prediction with experiment.

ii. Predict the direction and magnitude of the acceleration of the cart when you add 50 grams to the hanging weight and compare your prediction with experiment.
NEWTON’S LAWS (Part I)

Preliminary Questions

Name: ______________________________  Section: ____________
Date: _______________
Partner: _________________

1. Watch your language::
2. What is wrong with the following statement of Newton’s First Law?
   “If an object is moving with constant velocity, there is no force acting on it at all.”

2. A 6-kg mass is pulled along a frictionless track on a horizontal floor by a constant horizontal force T. The mass starts at rest and moves 3 m in 2.0 s. Draw a force vector diagram for the mass. What is the net force acting on the mass? What is the value of T?
NEWTON’S LAWS (Part II)

Preliminary Questions

Name: ______________________________  Section: ____________
Date: _________________
Partner: _______________

3. The block of 2 kg mass is placed on a smooth (no friction) incline. A $F = 5$ N force is required to keep it from sliding down the plane. For each of the following cases draw a force vector diagram and answer the following questions.

A. How large must $F$ be to keep the block moving up the incline at constant speed?
   What is the direction of $F$.

B. How large must $F$ be to accelerate the block from rest down the incline at $0.5$ m/s$^2$?
   What is the direction of $F$. 
Activity 1: Graphing.

A. What value of correlation R would you expect if you make a simple (linear fit) to your data? _________. What value did you obtain for R? _________. What do you conclude that R tells you about your data?

B. What value of correlation R would you expect if you make a simple (linear fit) to your data? _________. What value did you obtain for R? _________. What do you conclude that R tells you about your data?

What are the slope and y-intercept of the best-fit line? What would you expect them to be?

Include the graphs in your report.


Your assignment is to determine how $F$, $m$, and $a$ are related. You will use the setup sketched on page 3-4 of the manual. To keep the experiment simple, you first want to keep the ‘mass’ constant and vary the ‘force’ to see whether the ‘acceleration’ changes and how. The first step is to design the experimental procedure.

Before you do anything, consider the following questions:

What will you do with the two masses in this experiment, $m$ and $M$? Which mass do you want to keep constant and which mass will be varied? Or do you want to keep $m + M$ constant? Which of the forces is the actual $F$ that is relevant in Newton’s Second Law? How will you determine this relevant force? The force probe will measure the tension in the string. Is this tension the force $F$ in Newton’s Law?

The answer to above questions depends crucially on which ‘mass system’ you consider.

In the first experiment we will keep M (cart) fixed and vary m. In this case the net force on M is caused by the tension in the string. The tension varies as the weight m in the basket is varied. Clearly here the mass system you consider is the cart with fixed mass M.
Give a free body diagram of the system you are studying in this setup.

Make sure the track is perfectly horizontal. What goes wrong if it is not horizontal?

Make sure that the force sensor reads zero for zero force. (To calibrate, click on ‘Experiment’, then click on ‘Zero’, choose ‘force probe’. IMPORTANT CAVEAT: Only calibrate the force sensor while no forces act on the force sensor). The force probe is easily thrown off its calibration. Also note that its range switch is set correctly to 10 N.

Keeping the mass M constant, vary mass m and (using the motion sensor) measure the acceleration of the cart for FIVE different forces acting on M. For each case, record the mass M of the cart, the varied mass m of basket + weights (suggested is to use weights 20, 30, 40, 50, 60 gram for the FIVE cases). Record the value of the force, and the corresponding acceleration. For each choice of (M,m) make THREE runs and find the average acceleration a(average). Record all your measurements in a Table-format below.

<table>
<thead>
<tr>
<th>m-weights (kg) (+basket)</th>
<th>Force(N)</th>
<th>M-cart(kg)</th>
<th>a1(m/s^2)</th>
<th>a2(m/s^2)</th>
<th>a3(m/s^2)</th>
<th>a(average,m/s^2)</th>
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Plot the data using Logger Pro in Lab Apps and make a linear regression. Give your plot an appropriate title, label the two axes, identify the plot also with your name. Make a copy of the plot for your report. Interpret the meaning of the 2 constants (slope and intercept) in the linear regression. What is the coefficient of correlation?

What aspect of Newton’s Second Law has been addressed by this first experiment?

Can you interpret the slope in your graph in terms of the mass M(cart)?
List all forces acting on mass $M$ (include all horizontal and vertical forces). Represent all forces by arrows in a figure. For each force identify the origin/source (caused by whom) of the force, and the ‘nature’ (gravitational or non-gravitational) of the force.

Repeat this inventory for forces acting on mass $m$.

**End of Part I**
Activity 3: (1 hr) Fixed force
In this experiment we keep the force $F$ constant and the ‘mass system’ is $M + m$. The force is kept constant by fixing mass $m$ (weight + basket). Now the fixed gravitational force is $mg$ and this force acts on the system $M + m$, where we will vary $M$.

Record the value of the fixed force $F = mg$ in your experiment.
(Note that the tension is not relevant here, so ignore the reading of the force probe.)
We will measure the acceleration ‘$a$’ of the mass system $M + m$ for five different values of the cart mass $M$. $M$(cart) is varied by placing 0,1,2,3, or 4 metal bars (~500 gram each) on top of the cart.
Record your data in the Table below.
We will find the correlation between the mass of the system and its acceleration, while the force $F$ remains constant.

<table>
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<tr>
<th>F(N), fixed</th>
<th>Mcart(kg), variable</th>
<th>Mcart+weight+basket, “mass whole system”</th>
<th>Inverse system-mass</th>
<th>$a_1$(m/s^2)</th>
<th>$a_2$(m/s^2)</th>
<th>$a_3$(m/s^2)</th>
<th>$a$(average)</th>
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Make a graph of acceleration ‘$a$’ versus ‘inverse mass’, $1/(M + m)$ and make a linear digression. Report the correlation coefficient.
If this relation is linear, one can conclude that $(M + m) a = \text{constant}$ in this part of your experiment.

How does this relate to Newton’s Second Law?

The slope in your graph should tell you the value of the force $F$ you used. Why?
Calculate the value of the force $F$ from the parameters (slope and intercept) of your fit.
Activity 4: Predictions using Newton’s Second Law.
To test whether Newton’s second law is applicable to other situations, you tilt the track up at an angle of about 7° or less.
(Note: Finding the angle of inclination is done with triangulation. Measure ‘Base’ and ‘Height’ of the triangle that the track makes with a horizontal line parallel with the table top. Choose the length of the ‘Base’ as large as possible too have better accuracy for the angle)

First, determine the mass of the cart and determine the mass of the basket. MAKE SURE THAT THE CALIBRATION OF THE FORCE PROBE IS SUCH THAT A ZERO READING MEANS ZERO FORCE.

i. STATIC CASE:
Take mass m in the basket to be 70 gram and try to find the correct inclination for equilibrium by trial and error. Determine the angle by triangulation. Now reverse and starting with that angle, calculate what mass in the basket is needed for equilibrium (is it indeed close to 70 gram?). Show the calculation and include free-body diagrams for M as well as for m.

How would static friction affect the comparison? Which direction would the frictional force be?

By adding pennies to the 70 gram mass m, test by how much the mass m can be larger, while still maintaining equilibrium.

Test in the same way, by how much the mass m can be lowered while maintaining equilibrium. How do you interprete this range of forces? (HINT: without static friction the range would be zero.) How is the frictional force related to this range? Now, find the coefficient of static friction.
Adjust (with pennies) the mass $m$ to be in the middle of the allowable range for equilibrium, and measure the tension in the cord. (Crucial that the zero reading of the force probe is set correctly, ask your TA how.)

How is this tension related to the mass in the basket ($+m$ mass of the basket)?

How is this tension related to the mass of the cart?

From this tension find the angle of inclination. Is this angle more accurate than the angle that you found before from triangulation? Why?

ii. NON-STATIC: Predict the direction and magnitude of the acceleration of the cart along THE SAME inclined track when you add 50 g to the hanging weight. As angle of inclination, use the most accurate angle obtained above. Show your calculation, including a free-body diagram.

Do the experiment with the 50 gram added to the hanging weight and find the value of the acceleration $a$. Repeat this measurement 3 times, starting the cart each time at the same point, and average the value for the acceleration.

Compare experiment with your prediction.

(Do you need the reading from the force probe for this comparison?)
How would kinetic friction affect the experiment? Which direction would the frictional force be?

How large is the force of kinetic friction needed to bring experiment and your prediction into agreement? Is that reasonable? Which factors in your experiment determine the kinetic friction? Are these the same factors as for static friction?