Energy Extraction from Black Holes

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Special Relativity
Problem with Electricity and Magnetism

In the late 1800’s physicists realized there was a problem with electromagnetism: the speed of light was given in terms of fundamental constants.

\[
\frac{\partial^2 E}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 E = 0
\]

\[
\frac{\partial^2 B}{\partial t^2} - \frac{1}{\mu_0 \epsilon_0} \nabla^2 B = 0
\]

Is a problem because everything is independent of inertial reference frame and wave equation is not invariant under Galilean Transforms.

- Michelson-Morley Experiment - aether
Special Relativity

Galilean Transforms

Why Galilean Transforms:

- Newton’s Laws invariant under Galilean Transform
- Macroscopically make sense

\[ x \to x' = x + vt \]

\[ \frac{\partial^2 f}{\partial t^2} - c^2 \frac{\partial^2 f}{\partial x^2} = 0 \]

\[ \to \frac{\partial^2 f}{\partial t^2} - 2v \frac{\partial^2 f}{\partial x \partial t} - (c^2 - v^2) \frac{\partial^2 f}{\partial^2 x} = 0 \]
In the early-mid 1890’s famous physicist Lorentz figured out that the laws of electromagnetism are invariant under a different coordinate transformation:

\[
\begin{align*}
  x \to x' &= \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \\
  t \to t' &= \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}
\end{align*}
\]

Time and space are affected.
Special Relativity

Einstein’s Idea

Einstein figured out that Lorentz’s transformations could be derived from the principle of relativity and assuming that the speed of light is constant in all reference frames.

- For two observers at same location moving at different speeds, there is time dilation, $\Delta t' = \gamma \Delta t$
- For two observers at same time moving at different speeds, there is length contraction $\Delta x = \frac{1}{\gamma} \Delta x'$. 

(a) 

(b) 

\( \Delta t \) 

\( \Delta t_0 \) 

\( \Delta x \) 

\( \Delta t' \) 

Earth 

Alpha Centauri 

\( v \) 

\( L_0 \) 

\( L \)
Einstein realized that there was something special about gravity...
These ideas lead Einstein to propose ideas that drastically change the way we look at the universe:

- Time is not a parameter but another dimension like space - “Spacetime”
- Spacetime is dynamical - it is not flat, but is like a sheet of rubber that we live on
- Spacetime is curved by energy and mass:

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}
\]
Relativity also tells us that particles will move in “straight” lines called **Geodesics**. That is relative to the surface.
General Relativity
More Geodesics
The geodesics of this surface give keplerian orbits and gravitational lensing.
Black Holes
Solution for the Sun

Look for a spherically symmetric solution to Einstein’s field equations which cuts off at the radius of the star. Gives you solution for the Schwarzchild Black hole.
Black Holes
Schwarzchild Black Hole

- If a star’s radius becomes smaller than the **Schwarzchild radius** distance \( R_s = \frac{2Gm}{c^2} \), then it forms a black hole.
- Inside \( R_s \) nothing can escape the gravitational pull because the curvature becomes too great.
- There is a singularity of infinite mass density at \( r=0 \).

Anatomy of a Schwarzchild Black Hole

\[ R_s = \frac{2GM}{c^2} \]

- \( R_s \) is the Schwarzchild radius
- \( G \) is a gravitational constant
- \( M \) is the mass of the black hole
- \( c \) is the speed of light
There are 3 parameters which describe every type of black hole: M, Q, J

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<thead>
<tr>
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<th>Non-Rotating (J=0)</th>
<th>Rotating (J≠0)</th>
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<tbody>
<tr>
<td>Un-Charged (Q=0)</td>
<td>Schwarzchild</td>
<td>Kerr</td>
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<tr>
<td>Charged (Q≠0)</td>
<td>Reissner-Nordström</td>
<td>Kerr-Newman</td>
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Rotating black holes have many interesting properties

- Cause spacetime (**ergosphere**) nearby to corotate
- Often have **accretion disks**: a disk of superheated plasma around its equator like Saturn’s rings
- Are at the center of galaxies
Black Holes

Interstellar Black Hole: Gargantua
Rotating faster than the speed of light causes space and time coordinates to mix allowing for negative energy.
3 Main ways to extract energy from Black Holes

- Hawking Radiation - Quantum effect
- Blandford-Znajek Effect - Magnetohydrodynamic Effect
- Penrose Process - Exploitation of properties of Ergosphere
Strong curvature of spacetime causes spontaneous particles-antiparticle pair creation. There is a net flux of anti-particles into the event horizon which “evaporates” the black hole.
Rotating black hole with an accretion disk causes plasma to rotate, powering magnetic fields which swirl outwards, carrying angular momentum, particles, and energy.
A particle goes into the ergosphere of a rotating black hole and breaks into two pieces, one of which has negative energy. This falls into the black hole and the remaining leaves with a net gain in energy.
Summary

- Energy radiates from black holes naturally in two processes:
  - Hawking radiation - due to the quantum effects of intense curvature
  - Blandford Znajek effect - due to the corotation powering intense magnetic fields
- Energy can be actively extracted by using Penrose Process
  - Impractical
Questions?
Supplemental Material

Worm Holes