This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please submit the sheet with your bluebook, but you may keep the exam. No calculators/phones. Tackle any (simple) calculations on your own. Total credit 30 points.

1. (2 points each, 4 total.) Please be concise.
   (i) Which nuclear process is the source of helium on the sun? Which nuclear process is the source of helium on the earth?
   (ii) How is resonant absorption achieved in Mossbauer effect, i.e., how is the energy lost to recoil made negligible?

2. (1 point each, 10 total). True or false?
   i) Both SU(2) and SO(3) symmetries lead to angular momentum conservation.
   ii) The characteristic length of the nuclear force is one fm.
   iii) It is hazardous to collect large quantities of deuterium oxide because the deuterons in this heavy water will spontaneously fuse into helium (which is a very stable nucleus).
   iv) Just as the group SU(2) describes spin 1/2, group SU(2S+1) describes spin S.
   v) The Pauli matrices are hermitian.
   vi) The Pauli matrices are members of SU(2).
   vii) The Pauli matrices are generators of SU(2).
   viii) There is direct experimental evidence for proton decay.
   ix) There is direct experimental evidence for neutron decay.
   x) The $\beta$-decay is a two-body process.

3. (4 points.) Calculate the energy splittings for the S,P,D, and F levels due to a $-2\alpha L \cdot S$ term ($\alpha > 0$) in the nuclear potential. Be sure to indicate which level is lower in energy, e.g., when you consider $P_{1/2}$ and $P_{3/2}$.

4. (4 points.) A parent nucleus with a lifetime of 1.0 billion ($=10^9$) days decays to a daughter nucleus, which in turn decays with a lifetime of one day. Initially there $10^{23}$ parent nuclei...
in a sample, but no daughter nuclei. Approximately how many daughter nuclei are present after (a) 8.64 seconds (b) 1000 days? Notes: There are 86400 seconds in a day. Lifetime, not half-life. Differential equations not needed.

5. (4 points.) An example of a neutron-induced fission reaction is $n + ^{235}_{92}U \rightarrow ^{93}_{37}Rb + ^{141}_{55}Cs + 2n$. Using the mass values listed below, estimate the energy released in this reaction in MeVs. The masses are in atomic mass unit (amu), which is 931.5 MeV/c$^2$. Since calculators are not allowed, this simple problem is really an exercise in handling significant figures and approximate calculations.

\[ m_n = 1.0086649, M(^{93}_{37}Rb) = 92.9220328, M(^{141}_{55}Cs) = 140.9200440, M(^{235}_{92}U) = 235.0439231. \]

6. (4 points.) Consider an “equation of motion” $\nabla^2 \psi = -k^2 \psi$.

a) Does it obey the global U(1) gauge symmetry? (Consider $\psi \rightarrow \psi \exp(i\alpha)$, where $\alpha$ is a real number.)

b) Does it obey the local U(1) gauge symmetry? (This time make $\alpha$ a real function.)

c) This equation also fails to include an interaction (force). Incorporate a divergence-free force $\vec{F}$ ($\nabla \cdot \vec{F} = 0$) i.e., modify the equation of motion to make it obey local U(1) symmetry by including this interaction.

d) Verify that your modified equation obeys local U(1) gauge symmetry. It is ok to stop after showing that the phase term “goes through” the first derivative.