1) (15 pts) True or false?

i) Neutron is electrically neutral, but has nonzero magnetic dipole moment. This is an indication that it is not a fundamental particle but has an internal structure (involving distribution of charges). True.

ii) At Fermilab, strange particles are produced by hitting a target with protons. In such reactions, a strange quark is produced with an antistrange quark, thus conserving strangeness. True (strong interaction)

iii) A quark always carries a (nonzero) color charge. True. That is why we don’t see them.

iv) Baryons such as protons come in three colors. False. Stable particles are color singlets.

v) A gluon can couple to (i.e. interact with) itself. True. SU(3) is “non Abelian”

vi) A photon can couple to itself. False. U(1) is Abelian.

vii) Since the electron disappears in an Electron Capture (EC) reaction (where a nucleus grabs an inner shell electron thus lowering its Z by one unit), EC must be an EM interaction. False. Weak.

viii) The (electric) charge of an electron grows as you get closer to it. True. Q grows as you penetrate screening effect of virtual e+e- pairs.

ix) The color charge of a quark grows as you get closer to it. False. Asymptotic freedom tells you that the strong interaction gets smaller as distances decreases.

x) Heavy stable nuclei have more neutrons than protons. True, in order to dilute the coulomb repulsion.

xi) Existence of the positron prevents the self-energy of electron, i.e. the energy to assemble an electron, from blowing up. True. This is vacuum polarization.

xii) Higgs mechanism is needed to explain why the vector bosons of weak interaction are massive. True.

xiii) Higgs mechanism explains why we never find free quarks (or gluons). False, confinement
xiv) SU(3) has eight generators, which is why there are eight gluons in QCD. True.
xv) C.S. Wu’s Cobalt-60 experiment demonstrated parity violation. True.

2) (6 pts). (a) Explain the statement: “The curve of binding energy turns over.” Nuclei with low atomic number (A) have too much surface which results in a low binding energy trend. At the other end, high Z nuclei have too much Coulomb repulsion which also lowers the BE.

(b) Draw the Feynman diagram for electron-positron annihilation. See below.

(c) Draw the (quark level) Feynman diagram for the beta decay of a neutron. See below.

3) (7 pts) (Assume \( c = 1 \).) A \( \pi^0 \) with energy \( E \gg m_\pi \) travelling along the \( z \) axis decays into two photons at \( z = 0 \). The distance \( R \) between the two photons and the photon energies \( E_1 \) and \( E_2 \) are measured in an electromagnetic calorimeter located at \( z = z_0 \) (\( z_0 \gg R \)).

a) Prove the relation \( E_1 E_2 R^2 \approx m_\pi^2 z_0^2 \), which is used to identify the \( \pi^0 \). (Square the momentum equation \( \vec{p}_\pi = \vec{p}_1 + \vec{p}_2 \). Use \( E^2 = p^2 + m^2 \) and energy conservation to replace the momenta with \( E_1, E_2, m_\pi \) and the angle \( \theta \) between the photons. Then use \( \cos(\theta) \approx 1 - \theta^2/2 \) to replace \( \theta \) by \( z_0 \) and \( R \).

Soln: \[ p_\pi^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 \Rightarrow p_\pi^2 = p_1^2 + p_2^2 + 2p_1 p_2 \cos(\theta) \Rightarrow (E_1 + E_2)^2 - m_\pi^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos(\theta). \] Now cancel common terms, approximate \( \cos(\theta) \) and use \( \theta \approx R/z_0 \) to finish.

b) What separation \( R \) would you get in a calorimeter located 50 m away from a \( \pi^0 \) decay to two 10 GeV photons? (\( m_\pi \approx 135 \) MeV.)

Soln: \( (10)(10) R^2 = (0.135)^2 (50 \text{ m})^2 \) give \( R = 67.5 \) cm.

4) (6 pts) a) Find all (nine) products of two Pauli matrices and express your results compactly in terms of the Pauli matrices and the identity matrix. You will need to use \( \delta_{ij} \) and \( \epsilon_{ijk} \) as well.

\[
\begin{align*}
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{align*}
\]

b) Use your result to find the commutators \( (\sigma_i \sigma_j - \sigma_j \sigma_i) \) and anticommutators \( (\sigma_i \sigma_j + \sigma_j \sigma_i) \).
c) Show that for two vectors $\vec{a}$ and $\vec{b}$, $(\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \vec{a})$. $I$ is the 2x2 identity matrix.

It is easy to verify that $\sigma_x^2, \sigma_y^2$, and $\sigma_z^2$ all equal identity. In some sense, the Pauli matrices & the identity are four `quark parts' of the identity (unit). It is also easy to verify that $\sigma_x \sigma_y = i \sigma_z$ etc. All these products can be summarized as

$$\sigma^i \sigma^j = \delta^i_j + i \varepsilon^{ijk} \sigma_k$$

$$\Rightarrow [\sigma^i, \sigma^j] = 2 \varepsilon^{ijk} \sigma_k$$

$$\Rightarrow i \varepsilon^{ijk} \sigma_i = \sigma_i \sigma_j + \sigma_j \sigma_i$$

$$\Rightarrow (\vec{a} \cdot \vec{b}) = (\sigma^i a_i) (\sigma^j b_j) = \sigma^i \sigma^j (a_i b_j) = 2 \delta^i_j$$

$$= (\delta^i_j + i \varepsilon^{ijk} \sigma_k)(a_i b_j)$$

$$= a_i b_i + i \varepsilon^{ijk} \sigma_k a_i b_j$$

$$= (\vec{a} \cdot \vec{b}) + i \sigma^k (\varepsilon^{ijk} a_i b_j)$$

$$= (\vec{a} \cdot \vec{b}) + i \sigma^k (\vec{a} \times \vec{b})$$

$$= \vec{a} \cdot \vec{b} + i \overline{\sigma}^k (\vec{a} \times \vec{b})$$

5) (6 pts) Draw the Feynman diagrams for the extremely rare decays $\bar{K}_0 \rightarrow \pi^0 \nu \bar{\nu}$ and $K^- \rightarrow \pi^- \nu \bar{\nu}$. To do this, first identify every particles in the basic building block of these decays called the “Penguin Diagram” (see Figure) which tells you how the strange quark decays. You need to do only a little after that. $\bar{K}_0 = sd$ and $K^- = su$. (After the exam, figure out why the name “Penguin diagram”.)