This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please submit the sheet with your bluebook, but keep the exam. Calculators are not allowed, which means approximate calculations are encouraged.

1) (15 pts) True or false?
   i) Neutron is electrically neutral, but has nonzero magnetic dipole moment. This is an indication that it is not a fundamental particle but has an internal structure (involving distribution of charges).
   ii) At Fermilab, strange particles are produced by hitting a target with protons. In such reactions, a strange quark is produced with an antistrange quark, thus conserving strangeness.
   iii) A quark always carries a (nonzero) color charge.
   iv) Baryons such as protons come in three colors.
   v) A gluon can couple to (i.e. interact with) itself.
   vi) A photon can couple to itself.
   vii) Since the electron disappears in an Electron Capture (EC) reaction (where a nucleus grabs an inner shell electron thus lowering its Z by one unit), EC must be an EM interaction.
   viii) The (electric) charge of an electron grows as you get closer to it.
   ix) The color charge of a quark grows as you get closer to it.
   x) Heavy stable nuclei have more neutrons than protons.
   xi) Existence of the positron prevents the self-energy of electron, i.e. the energy to assemble an electron, from blowing up.
   xii) Higgs mechanism is needed to explain why the vector bosons of weak interaction are massive.
   xiii) Higgs mechanism explains why we never find free quarks (or gluons).
   xiv) SU(3) has eight generators, which is why there are eight gluons in QCD.
   xv) C.S. Wu’s Cobalt-60 experiment demonstrated parity violation.

2) (6 pts). (a) Explain the statement: “The curve of binding energy turns over.”
   (b) Draw the Feynman diagram for electron-positron annihilation.
(c) Draw the (quark level) Feynman diagram for the beta decay of a neutron.

3) (7 pts) (Assume c=1.) A π⁰ with energy $E \gg m_{\pi}$ travelling along the z axis decays into two photons at $z=0$. The distance $R$ between the two photons and the photon energies $E_1$ and $E_2$ are measured in an electromagnetic calorimeter located at $z = z_0$ ($z_0 \gg R$).

a) Prove the relation $E_1E_2R^2 \approx m_{\pi}^2z_0^2$, which is used to identify the π⁰. (Square the momentum equation $\vec{p}_{\pi} = \vec{p}_1 + \vec{p}_2$. Use $E^2 = p^2 + m^2$ and energy conservation to replace the momenta with $E_1$, $E_2$, $m_{\pi}$ and the angle $\theta$ between the photons. Then use $\cos(\theta) \approx 1 - \theta^2/2$ to replace $\theta$ by $z_0$ and $R$.)

b) What separation $R$ would you get in a calorimeter located 50 m away from a π⁰ decay to two 10 GeV photons? ($m_{\pi^0} \approx 135$ MeV.)

4) (6 pts) a) Find all (nine) products of two Pauli matrices and express your results compactly in terms of the Pauli matrices and the identity matrix. You will need to use $\delta_{ij}$ and $\epsilon_{ijk}$ as well.

b) Use your result to find the commutators $(\sigma_i \sigma_j - \sigma_j \sigma_i)$ and anticommutators $(\sigma_i \sigma_j + \sigma_j \sigma_i)$.

c) Show that for two vectors $\vec{a}$ and $\vec{b}$, $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = (\vec{a} \cdot \vec{b})I + i\vec{\sigma} \times (\vec{a} \times \vec{b})$. $I$ is the 2x2 identity matrix.

5) (6 pts) Draw the Feynman diagrams for the extremely rare decays $\bar{K}^0 \rightarrow \pi^0 \nu \bar{\nu}$ and $K^- \rightarrow \pi^- \nu \bar{\nu}$. To do this, first identify every particles in the basic building block of these decays called the “Penguin Diagram” (see Figure) which tells you how the strange quark decays. You need to do only a little after that. $\bar{K}^0 = s\bar{d}$ and $K^- = s\bar{u}$. (After the exam, figure out why the name “Penguin diagram”.)