1. (Prob. 5.3) See the Figure. After you draw the best visual fit line, its y intercept (i.e. t=0) gives $N_0 = 98.5$ counts/minute in $N = N_0 \exp(-\lambda t)$ and $\lambda = 0.0032$ per minute is obtained using the value at some large interval, say 400 min. The mean life is then $<t> = 1/\lambda = 5.2$ hr and the half life is $t_{1/2} = 0.693 <t> = 3.6$ hrs. It is a good fit because most points lie on the line and those which don’t are within error bars shown. (Optional: do a least-squares fit and find the chi-square goodness of fit for the six degrees of freedom available.)

2. (Prob. 5.4) Let $N = N_0 \exp(-\lambda t)$ be the number of $^{14}$C nuclei after time $t$, with $N_0 = (6.02 \times 10^{23}/12) \times 1.3 \times 10^{-12} = 6.5 \times 10^{10}$ the initial number of $^{14}$C nuclei. The activity at time $t$ is $4 \times 10^{-12} Ci = \lambda N_0 \exp(-\lambda t)$, giving $t = (1/\lambda) \ln(\lambda N_0/4 \times 10^{-12} Ci)$. Using $1 Ci = 3.7 \times 10^{10}/s$ and $\lambda = \ln 2/t_{1/2}$ one finds $t = 4300$ yr.

3. (Prob. 5.5) With $10^9$ gms of water and ten out of 18 nucleons in water being protons, there are a total of $(10/18) \times 6.02 \times 10^{23} \times 10^9 = 3.3 \times 10^{32}$ protons. Multiplying by $10^{-33}$ decays per year, we would expect roughly one decay every three years. Nothing will change by 2050 (except for the earth being a lot warmer) since the time elapsed is insignificant compared to the proton lifetime.

4. (Prob. 5.9) From the text, four hydrogens fusing into a helium release 24.68 MeV’s. Therefore, fusion of 1 gm hydrogen will yield $(6.02/4) \times 10^{23} \times 24.68$ MeV’s = $4 \times 10^{24}$ MeV’s, i.e. about seven times that from 1 gm of U235.

5. (Prob. 5.11) The numbers of nuclei satisfy the differential equations

\[ dN_1/dt = -\lambda_1 N_1, \]
\[ dN_2/dt = \lambda_1 N_1 - \lambda_2 N_2, \]
and
\[ dN_3/dt = \lambda_2 N_2, \]

with the initial conditions $N_1(0) = N_0$, $N_2(0) = N_3(0) = 0$. We expect $N_1$ to steadily decrease, while $N_2$ will initially increase as the decay from 1 produces it, but then decrease as the rate of decay from 1 decreases and the rate of decay of 2 into 3 increases. Since $N_3$ is stable it will increase as the decay of 2 produces it, and eventually it must approach $N_0$. 

The solutions of the differential equations with the given initial conditions are

\[ N_1 = N_0 \exp(-\lambda_1 t), \]
\[ N_2 = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} (\exp(-\lambda_1 t) - \exp(-\lambda_2 t)), \]

and
\[ N_3 = \frac{\lambda_2 N_0}{\lambda_1 - \lambda_2} \exp(-\lambda_1 t) + \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1} \exp(-\lambda_2 t) + N_0. \]

The Figures show the behavior of these expressions for \( \lambda_2 = 2\lambda_1 \) and \( \lambda_2 = \lambda_1/2 \). Note that the \( N_i/N_0 \)'s are plotted against \( \lambda_1 t \) and that the curves for \( N_1, N_2, \) and \( N_3 \) are red, blue, and green, respectively.

......... continued ............
Prob 11, \( \lambda_2 = \lambda_1/2 \)

Counts/minute

\[
y = 98.5e^{-0.0032x}
\]