1. (Prob. 4.2) (See Ch. 1 for impact parameter etc.) At $r_c$, the distance of closest approach for a head-on trajectory ($b = 0$), the initial KE equals the final Coulomb energy, giving

$$r_c = ZZ'\alpha \hbar c/E = 2 \times 92 \times 197.3\,\text{MeV}\,\text{fm}/(137 \times 4\,\text{MeV}) = 66\,\text{fm},$$

where $\alpha = e^2/(4\pi \epsilon_0 \hbar c) = 1/137$ is the fine-structure constant. This is much greater than the nuclear radius as well as the two given impact parameters. So the distance of closest approach will only be a bit larger than $r_c$ for the two cases. Since the centrifugal term falls off faster with $r$ than does the Coulomb one, the centrifugal term gets its “best chance” at the distance of closest approach $r_c$, where $V_C = E = 4\,\text{MeV}$. Recalling that the angular momentum can be written either as $L^2 = (pb)^2$, or as $L^2 = \hbar^2 l(l+1)$, the centrifugal term at this distance can be written as $V_L = \hbar^2 l(l+1)/(2Mr_c^2) = L^2/(2Mr_c^2) = (bp)^2/(2Mr_c^2) = E(b/r_c)^2 = E(b/66\,\text{fm})^2$. The centrifugal potential is therefore about 0.0002 and 0.01 times the Coulomb potential for $b = 1\,\text{fm}$ and $b = 7\,\text{fm}$ respectively. Again, using $L^2 = (pb)^2 = \hbar^2 l(l+1)$, $b=1\,\text{fm}$ corresponds to $l=0.87$ or $l=1$ and $b=7\,\text{fm}$ to $l=6$.

2. (Prob. 4.3) Because the proton is so much more massive than the electron (and neutrino), its recoil kinetic energy can be ignored to a first approximation, so that $Q = (m_n - m_p - m_e)c^2 = 782\,\text{KeV}$ is shared between the electron and neutrino, each having its maximum kinetic energy of $Q$ when the other has zero kinetic energy. The proton has its maximum kinetic energy when the electron-neutrino system has its smallest possible energy ($=m_e$) in the rest frame of the electron-neutrino system. This means that the neutrino is at rest, and given its negligible mass, can be ignored in any relevant frame. Back in the rest frame of the neutron, the proton and electron (+ ignored neutrino) must then have equal and opposite momenta $p$, and conservation of energy requires that

$$Q + m_e = p^2/(2mp) + \sqrt{m_e^2 + p^2}.$$

The proton’s kinetic energy is small and can be dropped, giving $p^2 = Q(Q + 2m_e)$ and then $T_{p_{\text{max}}} = Q(Q + 2m_e)/(2mp) = 782\,\text{KeV}(782\,\text{KeV} + 2 \times 511\,\text{KeV})/(2 \times 938.3\,\text{MeV}) = 0.752\,\text{KeV}$. 

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This confirms that the proton KE is small.

3. (Prob. 4.6) Assuming the alpha particle feels a potential of -40 MeV inside the nucleus, it will have a momentum of $p_i = \sqrt{2 \times (10 + 40) \text{MeV}/c^2 \times 4000 \text{MeV}} = 632 \text{MeV}/c$ inside the nucleus, and $p_o = \sqrt{2 \times 10 \text{MeV}/c^2 \times 4000 \text{MeV}} = 283 \text{MeV}/c$ outside. The wavelength inside the nucleus is then $\lambda_i = \frac{h}{p_i} = 2\pi \times 197.3 \text{MeV} \text{fm}/632 \text{MeV} = 2.0 \text{fm}$. For $^{12}C$, diameter=$2R = 2 \times 1.2 \times (12)^{1/3} \text{fm} = 5.5 \text{fm}$, so the alpha particle fits. The larger $^{238}U$ nucleus can fit many.

4. (Prob. 4.7) The mass formula, ignoring the pairing term, is:

$$B = a_1 A - a_2 A^{2/3} - a_3 Z^2/A^{1/3} - a_4 (A - 2Z)^2/A.$$  

Then

$$\partial B/\partial Z = -2a_3 Z/A^{1/3} + 4a_4 (A - 2Z)/A.$$  

Setting this equal to zero gives the value of $Z$ for the most stable nucleus for a given $A$:

$$Z = \frac{A}{2 + (a_3/(2a_4))A^{2/3}}.$$  

Taking $a_3 = 0.72 \text{MeV}$ and $a_4 = 23.3 \text{MeV}$ gives

$$Z = \frac{A}{2 + 0.015A^{2/3}}.$$  

$Z=125$ would be unstable due to an unpaired proton. $Z=126$ corresponds to $A=345$ and is feasible. It seems ok fission-wise with $Z^2/A=46$ (see next chapter). $Z=164$ ($A=478$) would fission with $Z^2/A=56$. There seems to exist an island of stability near $Z = 114$ and $N = 184$, giving $A = 298$. The formula above would say that the most stable nucleus with this $A$ has $Z = 298/2.67 = 112$, not too far from 114. A toroidal shape would let the protons avoid each other (at the expense of some increase in the surface term). The (attractive) nuclear force is short-range and works only on neighbors, but the (repulsive) Coulomb force acts at long distances.