1. (5 pts) True or false?
   i) Stern-Gerlach experiment (which showed spin quantization) employs a uniform magnetic field.
   ii) In the absence of external fields, $j$ is a good quantum number for the hydrogen atom.
   iii) Degeneracy pressure of neutrons prevents a neutron star from collapsing further due to gravity.
   iv) Band gaps are a consequence of spherical symmetry of potentials in crystals.
   v) Chemical potential in statistical mechanics comes from (the Lagrange multiplier used to ensure) conservation of matter, i.e. of the total number of particles.

2. (7 pts) Starting with the Schrödinger equation, derive the stationary-state wavefunctions and energies for a particle of mass $m$ in an infinite cubical (3 dimensions) well of dimension $a$. Show your work step by step, but you don’t need to normalize the wavefunctions. Find the degeneracies for the lowest five energy levels.

3. (6 pts) Obtain an estimate for the ground state energy for an infinite square well (1 dimension) potential using a second order polynomial as a variational trial wavefunction. Take the square well potential to be zero between $-a$ and $+a$, and infinite outside. Justify ignoring the Dirac-delta functions that you will encounter. How close in percent terms is your estimate to the exact answer? (hint: 10 approximates $\pi^2$ to 1.3%) 

4. (7 pts) An infinite square well has a perturbation in the form of a potential bump in the middle, i.e., the potential is zero for $-a < x < -b$, $V_0$ for $-b < x < b$, zero for $b < x < a$, and infinite otherwise. $(0 < b < a.)$ Calculate the energy shift due to the perturbation to the first order. Examine the result for as many special cases as you can. $(2cos^2(x) = 1 + cos(2x))$. 

5. (7 pts) Calculate the Fermi energy for “non-interacting” electrons in a two-dimensional infinite square well. Let $\sigma$ be the number of free electrons per unit area.

6. (4+4 pts)

a) For a spin-1 particle, calculate $S_x|1 \pm 1\rangle$, where $|s m\rangle$ denotes the conventional $S^2$ and $S_z$ eigenstate. Then construct the $S_x$ eigenstate that has zero eigenvalue. Assume $\hbar=1$. You will need: $S_x = (S_+ + S_-)/2$ and $S_\pm|s m\rangle = \sqrt{s(s+1) - m(m \pm 1)}|s (m \pm 1)\rangle$.

b) An electron is in the spin state $\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$. Normalize. Find expectation values and uncertainties for $S_x$, $S_y$, and $S_z$. Confirm consistency with the uncertainty principle.

Pauli matrices are: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.