Total credit 40 points. Do all problems and show all work. This is a closed book/notes exam, but a one-sided 8.5x11 sheet with only formulae is allowed. Submit the sheet with your bluebook. No calculators/phones, tackle any (simple) calculations on your own.

1. (5 pts) True or false?
   i) Stern-Gerlach experiment (which showed spin quantization) employs a uniform magnetic field.
   ii) When an extremely strong magnetic field is applied to the hydrogen atom, \( j \) is no longer a good quantum number.
   iii) Degeneracy pressure of neutrons prevents a white dwarf from collapsing further.
   iv) Band gaps are a general consequence of a periodic potential.
   v) In quantum statistical mechanics, the total energy constraint results in the concept of temperature.

2. (8 pts) Starting with the Schrodinger equation, derive the stationary-state wavefunctions and energies for a particle of mass \( m \) in an infinite cubical well of dimension \( a \). Show your work step by step, but you don’t need to normalize the wavefunctions. Find the degeneracies for the lowest five energy levels.

3. (8 pts) Use the variational principle to find an upper bound on the ground state energy for the quartic potential \( V(x) = x^4 \) using a gaussian trial wave function \( e^{-bx^2} \). Be sure to normalize first. \( \int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi} \), \( \int_{-\infty}^{\infty} x^2 e^{-x^2} \, dx = \sqrt{\pi}/2 \), and \( \int_{-\infty}^{\infty} x^4 e^{-x^2} \, dx = 3\sqrt{\pi}/4 \).

4. (9 pts) (a) Derive the lowest order correction (\( \langle \psi_n | \delta H | \psi_n \rangle \)) to the energy levels of the one-dimensional harmonic oscillator (\( V = (1/2)m\omega^2x^2 \)) due to the perturbation \( \delta H = \alpha p^2/(2m) \), where \( \alpha \approx 0 \). (b) Obtain the exact energy levels by recognizing that the perturbed Hamiltonian represents an oscillator with different \( \omega \). (c) Show that (a) follows from (b). \( p = i\sqrt{\hbar m\omega/2}(a_+ - a_-) \), \( a_+ \psi_n = \sqrt{n+1} \psi_{n+1} \), and \( a_- \psi_n = \sqrt{n} \psi_{n-1} \).
5. (10 pts) [There is little computation in this question, especially after part (a). You may assume $\hbar = 1$ for convenience. $|s m\rangle$ denotes the conventional eigenstates of $S_z$. A spin-2 particle is in the unnormalized $S_x$ eigenstate $|\psi\rangle = |2\, 2\rangle + 2|2\, 1\rangle + \sqrt{6}|2\, 0\rangle + 2|2\, -1\rangle + |2\, -2\rangle$.

(a) Use $S_x = (S_+ + S_-)/2$ and $S_{\pm}|2\, m\rangle = \sqrt{2(2+1) - m(m\pm1)}|2\, m\pm1\rangle$ to find the $S_x$ eigenvalue.

(b) Describe precisely the results of measuring (i) $S_x$, (ii) $S_x^2$, (iii) $S^2$ and (iv) $S_y^2 + S_z^2$.

(c) Using b(iv), what is $\langle S_x^2 \rangle$?

(d) (Normalize the wavefunction.) What are the possible measurement values for $S_z$, and what are their measurement probabilities?

(e) What are the possible measurement values for $S_z^2$, and what are their measurement probabilities?

(f) Using (d), calculate $\langle S_z \rangle$.

(g) Using (e), calculate $\langle S_z^2 \rangle$.

(h) How can the maximum $S_z^2$ (=4, from (e)) be larger than $S_y^2 + S_z^2$ (=2, from (b))?