1. (10 points.) Consider the wave function (for a fixed time $t = 0$) $\psi(x,0) = A(a^2 - x^2)$ for $-a < x < a$, and zero elsewhere. Determine (a) the normalization constant $A$ (b) expectations $<x>$, $<p>$, $<x^2>$, and $<p^2>$ (c) uncertainties $\sigma_x$ and $\sigma_p$, and (d) verify consistency with the uncertainty principle.

2a. (7 points.) (Assume $\hbar = m = 1$ in this problem.) Show that the wavefunction $xe^{-x^2/2}$ is an eigenstate of the harmonic oscillator. What is the energy? What is the probability of finding this oscillating particle outside its classical amplitude? You can leave your answer in terms of definite integrals involving gaussian functions, but then go home and evaluate the actual number.

2b. (3 points.) Uncertainty principle: If an electron is confined to a half-Angstrom region, roughly what would be its momentum and kinetic energy? (Electron mass=$(1/2)\text{MeV}/c^2$, 1Angstrom=$10^{-10}m = 10^5fm$, $\hbar c = 200 \text{(MeV)(fm)}$, $1fm = 10^{-15}m$, and $1\text{MeV} = 10^6eV$. Maybe you recognize what this is, but ok if you don’t.)

3a. (7 points.) Consider the (unnormalized) wavefunctions $\psi_a = \psi_1 + \psi_2 + \psi_3$ and $\psi_b = \psi_1 - 2\psi_2$, where the three $\psi_i$’s form an orthonormal basis. (a) Explicitly calculate the projection operator (matrix) along $\psi_b$. (b) “Make” $\psi_a$ orthogonal to $\psi_b$, i.e. extract the part of $\psi_a$ that is orthogonal to $\psi_b$ using the projection matrix that you just calculated. (c) Verify the orthogonality.

3b. (3 points.) Show that $[x,p] = i\hbar$, where $x$ and $p$ are the usual position and momentum operators, respectively. Then use this result to evaluate $[x,p^2]$ without explicit differentiation. (Think “add/subtract terms”.)