Assessing Student Performance in Solving Complex Physics Problems

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Abstract

In physics, students are taught to solve complex problems by (1) decomposing the problem, (2) recognizing the physics principles that are involved, (3) instantiating the appropriate equations for each principle, and (4) combining the equations to form a system of equations that can be used to solve for the value of a variable. To assess the understanding of the student, a system must be able to use the student's equations to determine which principles the student has either learned incorrectly or has omitted from the answer. This paper describes a method of analysis and assessment based on (1) representing the domain knowledge that is used in a problem, and (2) reasoning from the student's answer and the knowledge to provide guidance as to the student's next step. The method solves some difficult issues and an initial implementation has been successfully tested on a small number of problems from Newtonian mechanics.

Introduction

Central to any quantitative introductory physics course is the requirement that students solve complex problems in which physical situations are described in natural language. To solve a problem, the student must analyze the problem statement to find the general physics principles that apply, write the equations that follow from applying the principles to the problem, and use mathematical methods to solve the equations. Prior to providing help for such a student, a tutoring system must first assess the student's understanding of the physics principles involved, including a determination of the parts of the problem that are troubling the student. One of the key pieces is to determine the principles the students have mastered and the principles the students are having trouble with.

Many tutoring systems have been developed that are based on evaluating only the correctness of answers. Such approaches can be effective when each question is associated with a single principle but are ineffective in isolating misconceptions when the questions involve the comprehension and application of multiple principles. Many exercises in physics are designed to test whether students have learned how to (1) decompose complex problems, (2) recognize the principles used in the problems, (3) instantiate the appropriate equations, and (4) combine the solutions to individual subproblems into a solution for the whole problem. When provided with student solutions to such exercises, many current assessment tools are unable to identify the underlying misconceptions and cannot assess a student's mastery of the individual topics.

This paper describes a method for assessing a student's understanding based on the system of equations she specifies when solving a complex problem. The method is based on (a) specifying the knowledge that is used in a problem, (b) constructing data structures that support easy and efficient analysis of student answers and (c) heuristic techniques for reasoning from these structures to determine the student's mastery of the physics principles. An initial assessment of the method has been performed with successful results on several classes of problems in Newtonian mechanics from introductory physics courses.

An Example Problem

An example of a complex problem from introductory physics is that of a collision between two objects. The following description is one of the many variants of the problem and is shown in Figure 1.

> A bowling ball collides head-on with a stationary pin. Assuming that the collision is elastic, give a set of equations that are sufficient to solve for the final velocity of the pin.

Figure 2 contains a solution to this problem represented as a set of equations each of which is closely related to the physics principles relevant to the problem. Equations 1 through 7 deal with conservation of momentum in the system while equations 8 through 14 deal with conservation of energy.

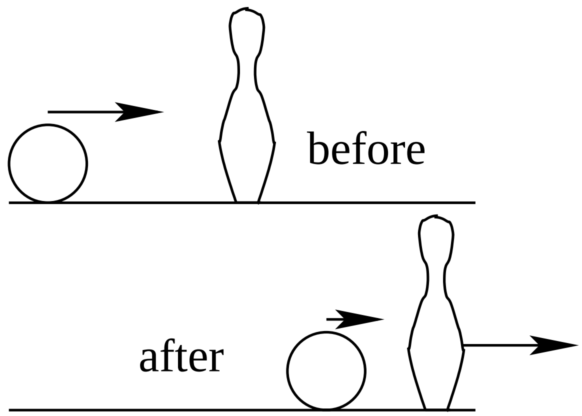

Figure 1: Collision Between a Bowling Ball and a Pin

$$p_{b,I} = m_b * v_{b,I} \quad (1)$$
$$p_{p,I} = m_p * v_{p,I} \quad (2)$$
$$p_{b,F} = m_b * v_{b,F} \quad (3)$$
$$p_{p,F} = m_p * v_{p,F} \quad (4)$$
$$p_{T,I} = p_{b,I} + p_{p,I} \quad (5)$$
$$p_{T,F} = p_{b,F} + p_{p,F} \quad (6)$$
$$p_{T,I} = p_{T,F} \quad (7)$$

$$E_{b,I} = 1/2 * m_b * v_{b,I}^2 \quad (8)$$
$$E_{p,I} = 1/2 * m_p * v_{p,I}^2 \quad (9)$$
$$E_{b,F} = 1/2 * m_b * v_{b,F}^2 \quad (10)$$
$$E_{p,F} = 1/2 * m_p * v_{p,F}^2 \quad (11)$$
$$E_{T,I} = E_{b,I} + E_{p,I} \quad (12)$$
$$E_{T,F} = E_{b,F} + E_{p,F} \quad (13)$$
$$E_{T,I} = E_{T,F} \quad (14)$$

$$v_{p,I} = 0 \quad (15)$$

Figure 2: Complete set of equations describing an elastic collision between a ball and a pin in one dimension.

Student answers are typically quite different in style, number of variables, and number of equations than what is shown in Figure 2. A student usually combines equations algebraically and also uses different variables. An example of a correct answer is given by Equations 16–17. To arrive at this answer, the student used v_b in place of $v_{b,I}$, $v_b^{'}$ in place of $v_{b,F}$, $v_p^{'}$ in place of $v_{p,F}$, and has eliminated terms involving $v_{p,I}$ because the pin is initially at rest. She combined equations 1–7 and 15 to form equation 16, and combined equations 8–14 and 15 to form equation 17.

$$m_b * v_b = m_b * v_b' + m_p * v_p' \tag{16}$$

$$1/2 * m_b * {v_b}^2 = 1/2 * m_b * {v_b'}^2 + 1/2 * m_p * {v_p'}^2 \tag{17}$$

This answer is typical of correct answers provided by students. However, when a student provides an incorrect answer such as that shown in equation 18 (where the student erroneously combined equations 1, 2, 5 and 15), the process of identifying the physics principles that the student misunderstood or misused is quite challenging. In equation 18, the student has incorrectly chosen to omit conservation of energy and furthermore has neglected to consider the momentum of the ball after the collision. In addition, she has correctly used equation 15 to simplify the equation by omitting the initial momentum of the pin since the initial velocity of the pin (v_p) is zero.

$$m_b * v_b = m_p * v_p' \tag{18}$$

In previous work [6, 7, 8, 9, 10, 11], we described techniques that determine the dimensions of each variable in a system of equations and furthermore can map each equation to an element of a solution set of equations. For the rest of this paper, we assume that the dimensions of the student equations and variables have been determined and correctly mapped to equations and variables from a solution set.

Previous Work

Physicists have been at the forefront in using computers to help students in the process of problem solving and in evaluating the results, but such systems are still quite primitive. The most common systems now are the numerous homework generating and grading systems, usually web-based, which randomize numerical values in a set problem, and can read the student's numerical, multiple choice or (in some cases) symbolic answer, and mark it right or wrong. Examples of such systems are WebAssign[18], CyberProf[5] and WeBWorK[1]. These systems inform the student whether her final answer is correct, but otherwise give little or no guidance and make no effort to evaluate the cause of mistaken results.

Somewhat more ambitious is *Mastering Physics*[12], Addison-Wesley's adaptation of the CyberTutor[13] system from MIT. This allows a problem author to include hints and fall back subproblems, to provide the student who cannot solve the original problem with some pre-authored tutoring. The problem author may also recognize known mistakes and provide appropriate responses. These systems do not decompose problems down to fundamental principles, however, so while they can assess how difficult each subpart of the question is, they cannot analyze which aspects of physics comprehension is at fault. Another system, the “Personal Assistants for Learning”[14] of CIRCLE, can lead the student through a tightly constrained path of multiple choices, providing suitable pre-authored responses.

A much more ambitious approach builds general physics principles as well as methods of utilizing these principles into a knowledge base. Specific problems are introduced to these systems simply by stating them in a formalized way, and the system can analyze the problem and solve it in a way similar to (although more explicit than) the way a human would. Because these systems “understand” the solution to the problem, they can analyze student solutions and attribute difficulty to specific lacks of understanding in the student. The most advanced such system is ANDES[2, 15, 16], although efforts go back to the 70’s with PLATO[17]. ANDES originally attempted to evaluate the probability that a student understood each “problem solving method” that could be used to solve a problem, based on a Bayesian analysis of her answers[4].

Analysis of the Student’s Answer

There are many physics principles and problem solving steps that are required to generate the system of equations in Figure 2. The student must

- realize that conservation of momentum and conservation of energy should be applied to a closed system,
- define a closed system of objects,
- understand the relationship between the total momentum and energy of a system and of the individual objects that comprise the system,
- know the definitions of momentum and energy for an object, and
- apply these physics principles to generate the system of equations describing the system at hand.

A student may have difficulties with the principles or with problem solving steps. There are several categories of errors or mistakes that a student can make. Among them are:

Failure to apply a necessary principle

The student does not identify a principle that is applicable to the problem. The consequence is that some equations are omitted. A common mistake is for students to not realize that energy is also conserved in elastic collisions and, thus, to omit the equations dealing with conservation of energy.

Use of a non-applicable principle

> The student incorrectly identifies and applies a principle resulting in an incorrect (additional) equation. A common mistake is for students to apply energy conservation to an inelastic collision and, thus, have the additional equations dealing with energy and its conservation. The last of these is inconsistent with the correct equations, while the others are correct but irrelevant.

Incorrect application of a necessary principle

> The student correctly identifies the principles that are involved but incorrectly applies them resulting in incorrect equations. For example, terms may have incorrect signs, or incorrect projections if the quantities involved are vectors.

Algebraic or arithmetic errors

> Students frequently combine equations algebraically (see equations 17 and 18). This does not cause difficulties if the combination is performed correctly. However, a mistake in combining equations may result in an equation that either has incorrect dimensions or manifests itself as an error of a different type. It is difficult to determine the actual error in these circumstances. Providing feedback for these errors is described in [6, 7].

Irrelevant but correct equations

> Students frequently write down all the equations that they know without regard to whether they are required to solve the problem. For example, if the problem involved an inelastic rather than an elastic collision, the students can write equations that define the energy of the ball, the pin and the energy of the combination both before and after the collision. As long as there is no equation applying conservation of energy, these additional equations would be considered correct but irrelevant and therefore would not be considered as errors in misunderstanding or misapplying the principles, although they indicate uncertainty in how to approach a solution.

A tutoring system should recognize and process most if not all of these types of errors. In this paper we assume that algebraic or arithmetic errors are detected and handled by a separate procedure (see [10, 11]). We focus primarily on the area of detecting either missing or additional equations where the errors indicate a misunderstanding or mis-application of the physics principles.

We use an annotated solution to build a solution structure showing how the equations relate to one another, the physics principles used to generate each equation and the time intervals during which events occur. The annotations specify (a) the principles, (*e.g.*, momentum conservation), (b) the objects, *e.g.*, a system of a ball and a pin, (c) the event, *e.g.*, elastic collision and (d) the time instant of the event, *e.g.*, t_2. The system parses these annotations and builds a graph of the equations annotated with links based on domain knowledge of the components specified in (a), (b), (c), and (d). A complete abstracted (without annotations) version of the solution graph corresponding to our example problem is shown in Figure 3. The figure shows how the equations are linked together by the definition of a variable in one equation and its subsequent use in other equations. The annotations in the graph link (1) the variables to properties of objects and systems of objects, and (2) the equations to the physics principles underlying the equation.

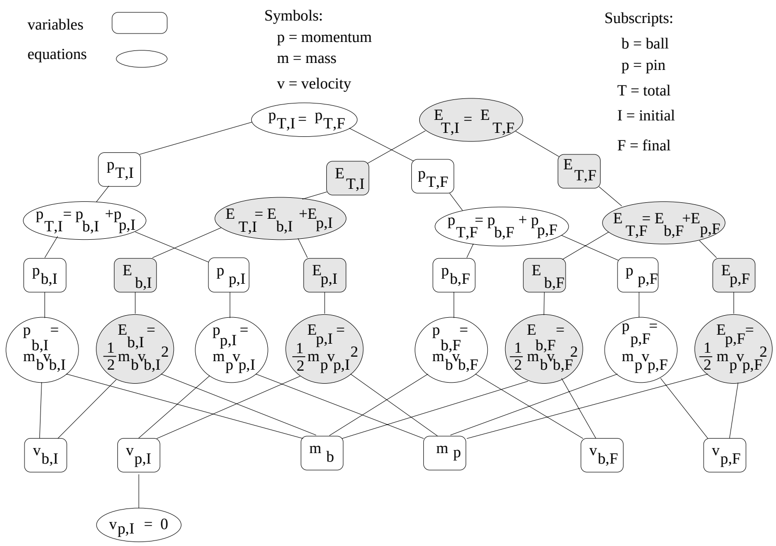

Figure 3: Solution graph for the Collision Problem

Figure 4 shows how the annotated physics principles would be linked to individual equations in part of the solution graph.

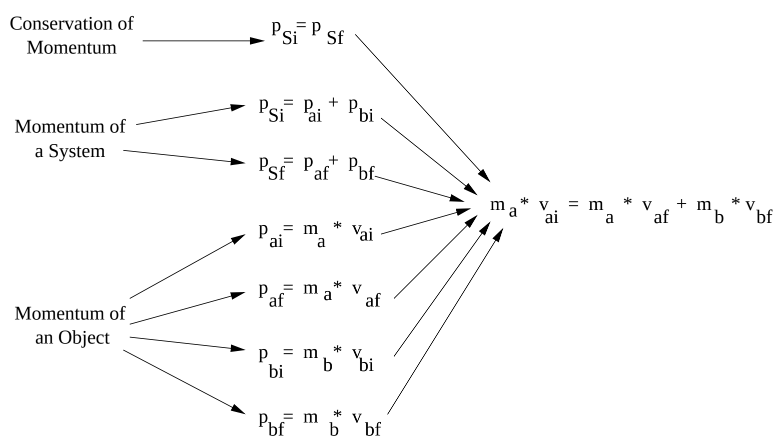

Figure 4: Momentum Component of the Solution graph for the Collision Problem

With these structures and a heuristic mechanism for mapping student equations to equations in the solution set, students equations can be analyzed to determine if they are: (a) correct and relevant, (b) incorrect but mapped to an equation, (c) correct but irrelevant and therefore not mapped to any equation, and (d) incorrect and extraneous, *i.e.*, not mapped to any equation. In addition, if there are equations in the solutions set that do not map to a corresponding student equation, those equations are marked as missing. From this analysis, it is a straightforward matter to generate feedback for the incorrect equations ((b) above) (see [11]).

Analysis of missing equations is more problematic, specifically when more than one equation is missing. The simple approach of selecting a single missing equation and assuming that the student does not understand the principle used to generate the equation is unsatisfactory if there are several missing equations that could be linked to a higher order principle. A good tutoring system would take the analysis one step further to determine the commonality, if any, underlying the problematic principles. Consider the case where the student had generated the equations for momentum and energy of the ball and the pin but left out all other equations. An initial analysis would indicate that the student had difficulty, separately, with the principles of conservation of energy and conservation of momentum. Additional heuristics can recognize that the two missing principles both require that the ball and pin form a *closed* system, and that is what the student needs help on.

The heuristic technique analyzes the solution graph with the following algorithm to generate hints for the student. All equations in the graph that can be mapped to student equations are marked as PRESENT. The algorithm then traverses the structure to find candidate nodes and their associated techniques at which to hint. For example, if a marked node has unmarked children, the student may understand a concept but not how to implement it, and a hint at one of the unmarked children may be given. Another candidate is a high unmarked node with most of its descendants unmarked, indicating a concept whose use is unrecognized by the student, and the high level node (and principle) should be hinted at. A third possibility is an unmarked node all of whose children are marked, an obvious candidate for hints.

Heuristics are used to select the most appropriate hint based on the information gathered by the algorithm. The information also includes a chronological timeline showing the order in which the equations are generated as this gives some idea as to what the student was last working on. For example, if the student was working on instantiating a principle it would not be helpful to point the student in another direction.

Presented with an answer consisting only of equations 1–7 and 15, the system recognizes that equations 8–14 are missing. From examination of the solution graph, it detects that equation 12 is an ancestor of equations 8 and 9, equation 13 is an ancestor of equation 10 and 11, and equation 14 is an ancestor of equations 12 and 13. Equation 14 is thus the highest common ancestor with an annotation of “Energy Conservation of A System”. This analysis can be used for many purposes. An assessment system would use the analysis to determine the concepts that are understood or misunderstood. A tutoring system would additionally be able to generate useful guidance for the student. For example, our system generates guidance suggesting to the student that “energy is also conserved in this problem”. A template of increasingly more specific help is used to generate the guidance. These templates are indexed by the physics principle that is involved. For our example, the system generates in succession (if the student repeatedly seeks help for the same problem) guidance stating that (1) the student consider energy conservation, (2) the student consider the system for which energy is conserved, (3) the system consists of the ball and the bin and finally (4) the actual missing equation (equation 14).

An initial assessment of our system was undertaken on collision problems involving (1) closed systems, (2) conservation of momentum and in some cases (3) conservation of energy. These are problems that have proven to be particularly difficult for previous approaches (see [4, 3]) because of (a) the interactions of the principles and (b) the notion of a single closed system is applied to many equations. Case (b) is particularly interesting because it only applies if the student has a common error in the equations generated, e.g., all the conservation equations are missing the terms involving the pin. In all cases where a single principle caused errors, the system was able to correctly determine the faulty principle even when multiple equations were involved. Our system was also able to correctly diagnose some failures caused by multiple interacting principles. Though the techniques are unable to handle all problems arising from multiple failures, we believe the techniques can be further developed to provide effective coverage for multiple failure problems.

Issues arise from the contributions of physics principles that are not directly related to any single equation. For example, conservation of momentum and conservation of energy both apply to closed systems. Each conservation principle leads directly to an equation while the principle that determines the ball and pin form a closed system is not directly linked to any one equation. An error with both the conservation of momentum and conservation of energy equations could be caused by difficulties in defining a closed system. Without an understanding of the relationship between the two conservation equations, an assessment system would only be able to critique the individual equations and not the overall set. Our system is able to identify errors arising from the incorrect definition of a closed system (the common principle) but had problems identifying incorrect application of conservation principles.

Conclusion

Assessment of a student's understanding of concepts and principles from their answers to problems becomes increasingly difficult as problems become more complex. Physics is an example of a domain where the problems are typically complex and a correct assessment is much needed. We have described a technique for analyzing and assessing student answers in the form of systems of equations. The technique uses annotations of a given solution that refer to pieces of domain knowledge that are contained in the system. The annotations and domain knowledge are used to construct a solution graph with annotations. Heuristic techniques are then used to analyze the student's answers and determine the principles that were misapplied or misunderstood. An initial implementation on a small set of problems has been successful at solving difficulties that have proven to be problematic for existing systems. Much more work remains to be done to extend the method to a larger set of problems.

References

[1] GAGE, M. E., AND PIZER, A. K. WeBWorK – math homework on the web. In *Electronic Proceedings of the Annual International Conference on Technology in Collegiate Mathematics* (1999).

[2] GERTNER, A., AND VANLEHN, K. Andes: A coached problem solving environment for physics. In *Proceedings, 5th International Conference, ITS 2000* (Montreal Canada, June 2000), Springer.

[3] GERTNER, A. S. Providing feedback to equation entries in an intelligent tutoring system for physics. In *Proceedings of the 4th International Conference on Intelligent Tutoring Systems* (1998).

[4] GERTNER, A. S., CONATI, C., AND VANLEHN, K. Procedural help in andes: Generating hints using a bayesian network student model. In *Proceedings of the 15th National Conference on Artificial Intelligence* (1998).

[5] HUBLER, A. W., AND ASSAD, A. M. Cyberprof: An intelligent human-computer interface for asynchronous wide area training and teaching. In *Proceedings of Fourth International World Wide Web Conference* (1995).

[6] LIEW, C., SHAPIRO, J. A., AND SMITH, D. Reasoning about algebraic answers in physics. In *Proceedings of Twelfth International Florida AI Research SocietyConference* (1999), pp. 167–171.

[7] LIEW, C., SHAPIRO, J. A., AND SMITH, D. What is wrong with this equation? error detection and feedback with physics equations. In *Proceedings of Thirteenth International Florida AI Research SocietyConference* (2000).

[8] LIEW, C., SHAPIRO, J. A., AND SMITH, D. Identification of variables in model tracing tutors. In *Proceedings of 11th International Conference on AI in Education 2003* (July 2003), IOS Press, pp. 464 – 466.

[9] LIEW, C., SHAPIRO, J. A., AND SMITH, D. Inferring the context for evaluating physics algebraic equations when the scaffolding is removed. In *Proceedings of Seventeenth International Florida AI Research Society Conference* (2004).

[10] LIEW, C., AND SMITH, D. Checking for dimensional correctness in physics equations. In *Proceedings of Fifteenth International Florida AI Research Society Conference* (2002).

[11] LIEW, C. W., AND SMITH, D. E. Reasoning about systems of physics equations. In *Intelligent Tutoring Systems: ITS 2002* (2002), Cerri, Gouarderes, and Paraguacu, Eds., Lecture Notes in Computer Science, Springer-Verlag. (LNCS 2363).

[12] `http://www.masteringphysics.com/`.

[13] PRITCHARD, D. E., AND MOROTE, E.-S. Reliable assessment with cybertutor, a web-based homework tutor. In *World Conference on E-learning in Corporate, Government, HealthCare, and Higher Education* (Montreal, Canada, 2002), E-Learn.

[14] REIF, F., AND SCOTT, L. A. Teaching scientific thinking skills: Students and computers coaching each other. *Am. J. Phys 67* (1999), 819–831.

[15] SCHULZE, K., SHELBY, R., TREACY, D., WINTERSGILL, M., VANLEHN, K., AND GERTNER, A. Andes: An intelligent tutor for classical physics. *The Journal of Electronic Publishing 6*, 1 (2000).

[16] SHELBY, R. N., SCHULZE, K. G., TREACY, D. J., WINTERSGILL, M. C., VANLEHN, K., AND WEINSTEIN, A. An assessment of the Andes tutor. In *Proceeding of the 5th National Physics Education Research Conference* (Rochester, NY, July 2001).

[17] SHERWOOD, B., AND STIFLE, J. The PLATO IV communications system. Tech. rep., University of Illinois Computer-based Education Research Laboratory, Urbana, IL, 1975.

[18] TITUS, A. P., MARTIN, L. W., AND BEICHNER, R. J. Web-based testing in physics education: Methods and opportunities. *Computers in Physics 12, #2* (1998).