

What Is Wrong With This Equation?

Error Detection and Feedback with Physics Equations

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Abstract

One of the keys to a good Intelligent Tutoring System (ITS) is its ability to identify and localize the error in a student's answer and then generate useful feedback. This paper describes an algorithm that we have developed for analyzing errors in algebraic equations and generating feedback that is specific to the context. We have implemented this algorithm and identified some of the knowledge that is useful for this domain. Our experiments indicate that many errors can be handled with a limited amount of domain knowledge.

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Introduction

One of the keys to the success of “good” human tutors is their ability to identify and localize the error in a student's answer and then generate useful feedback. Useful feedback enables students to clear up their mistakes or conceptual errors without directly giving them the answer. The degree to which an Intelligent Tutoring System (ITS) can (1) pinpoint an error in the answer, (2) identify the conceptual errors that led to the mistake and (3) analyze the problem context to generate feedback, greatly impacts on the success of the system. This credit-blame assignment problem has proven to be difficult to solve.

This paper describes our work in addressing the credit-blame assignment problem in the context of a tutoring system for an introductory college level Physics course. We have developed and implemented an algorithm for analyzing errors in algebraic equations and generating feedback that is specific to the context. We have identified some of the knowledge that would be necessary for generating useful feedback in this specific domain. We have categorized the types of errors that can occur in a set of such equations and have found that many of the errors can be handled with a limited amount of domain knowledge.

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An Example Problem

A basic concept taught in elementary physics is that of **Force Balance** in statics. There are many variations and contexts in which this concept is applied. One common example is that of a block *A* on an inclined plane that is connected via a pulley to another block *B* (see Figure 1). If the blocks are not moving, then the tension which balances the force of gravity on block *B* also balances the resultant of the other forces on block *A*. Students are frequently asked to specify the equation relating the forces acting on the masses. One of the several possible correct answers is:

$$m_A * g * \sin(\theta) = m_B * g$$

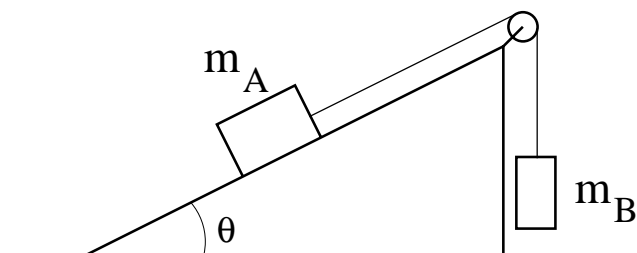


Figure 1: Example Problem: Blocks and Inclined Plane

The objective of this example problem is to determine whether students:

- understand that in equilibrium the net force acting on each block separately must vanish,
- can find equations from that principle *and*
- can put these together to find a single equation for the masses and θ .

The various quantities, *e.g.*, Mass and Acceleration, are given as variables to help the student learn how

to reason abstractly, *i.e.*, without relying on numeric quantities and calculators.

Some example answers are listed below:

1. $m_A * g * \sin(\theta) - m_B * g = 0$
2. $g * (m_A * \sin(\theta) - m_B) = 0$
3. $m_B * g + m_A * g * \sin(\theta) = 0$
4. $m_B * g - m_A * g * \cos(\theta) = 0$
5. $m_A * g = 0$

The answers above are all dimensionally consistent, *i.e.*, all the terms are Forces, but only the first two are correct. The rest are incorrect in different ways, and sometimes in a combination of ways. The issues we are interested in are (1) how to detect and localize the errors and (2) how to generate appropriate feedback to the student. In an earlier paper (Liew, Shapiro, & Smith 1999), we described how we have approached the problem of making sense of the equation in terms of physical quantities, *i.e.*, how do we recognize that $m_A * g$ is a Force?

Some Current Approaches and Techniques

There are several techniques that have been used for the identification of conceptual errors and generation of appropriate feedback. These techniques include:

- allowing the user to only select from a list of pre-enumerated responses: Each response is associated with a known conceptual error. Multiple choice mechanisms are an example of this technique.
- enumerating the possible answers and comparing the submitted answer against the enumeration: Each member of the pre-enumerated list would have to be matched against a corresponding message or a template for a message. This can be painstaking and require much effort on the part of the problem designer.
- simulation of the submitted answer: The tutoring system incorporates either implicitly or explicitly a model of the concepts being tested. The student's answer is simulated to determine what (incorrect) outcomes could be generated. The CIRCSIM (Hume *et al.* 1993; 1995) system uses a knowledge base of causal models for tutoring medical students on cardiovascular physiology.
- The ANDES system (VanLehn 1996) compares each term in an algebraic equation against terms in the solution. Our approach is similar but takes a further step in generating feedback that is specific to Physics. The ANDES system's feedback is more general and refers more to algebraic errors.

The Credit-Blame Assignment Problem

The difficulty in analyzing student answers affects our ability to provide helpful feedback on incorrect answers beyond the terse and unhelpful "Your answer is incorrect". This difficulty relates to some of the earliest work in building intelligent systems, that of credit-blame assignment. Credit-blame assignment in the context of an intelligent tutoring system consists of the following steps:

- determine that the submitted answer is incorrect.
- localize the error to a component of the answer.
- identify the likely conceptual error or mistake that led to the mistake.
- generate feedback based on the context of the problem, *i.e.*, what is the nature of the correct answer versus the submitted answer? What information can be provided that will guide the student without directly giving them the answer.

We have developed an algorithm for performing credit-blame assignment for algebraic answers in Physics. Our algorithm is strongly biased towards the domain of introductory Physics because it uses strong domain knowledge to reason about the answers and perform credit-blame assignment. The algorithm has been implemented in the PHYSICS-TUTOR system, an ITS that is being developed for introductory Physics courses.

Error Detection

How then do we perform credit-blame assignment for algebraic equations in an introductory Physics course? We start by restricting the kinds of answers that the student can provide.

- All variables required to solve the problem are explicitly mentioned in the problem statement or are well-known standard quantities (*e.g.*, g is the standard gravitational constant at Earth's surface). The instructor provides information about the variables that cannot be deduced by the system. This information includes the dimensions of the variable and where possible the Physics concept (*e.g.*, Force, Mass) represented by the variable.
- A unique identification of objects is required to localize and point out errors to the student. The system has heuristics about the features that can be used to uniquely identify the objects in the problem. Currently, the heuristics use properties of objects to identify the objects. The properties of objects are sometimes unique, *e.g.*, mass m_1 for an object, but

sometimes are not, *e.g.*, objects A and B may both have mass m_1 but may have unique velocities v_0, v_1 .

Given the above restrictions and a correct answer (also an algebraic expression) supplied by the instructor, the PHYSICS-TUTOR system uses the following algorithm to analyze the student's submitted expression:

1. Transform the submitted expression into a canonical form. The canonical form is a *sum of products* = 0.
2. Map all variables in the expression to dimensions and determine the dimensions of all the terms in the expression tree. If the expression is not dimensionally consistent or if the expression contains an unknown variable, then generate an appropriate error message to the user. If the expression contains an unknown variable, the user has the option of specifying the dimensions of the variable, as well as modifying the submitted answer.
3. Map terms in the expression to terms in the answer. Each term in the expression is matched against every term in the answer using a heuristic based on the symbols that occur in each term. There is a minimum level of match beyond which terms are considered not to match. The best overall matches are then performed first and the process iterates until all matches have been made. Any terms left unmatched are either missing or extraneous terms.
4. Compare every term in the answer to the corresponding term in the submitted expression. This step localizes errors to terms in the answer and also identifies the specific errors in the expression.

The result of the overall comparison is that the submitted expression (student's answer) is classified as one of the following:

- Correct expression.
- Incorrect expression but close to the answer. There is either a single error or a combination of two errors that caused the expression to be incorrect. A response (message) is then generated to help the student correct their answer.
- Incorrect expression. There are more than two errors in the expression. The system concludes that there is a lack of conceptual understanding on the part of the student. The system then tries to decouple the components of the problem. For example it could ask for the initial set of equations from which the solution was obtained or propose an explicit version

of the problem that employs a freebody diagram of one of the blocks. Hopefully, such suggestions will focus attention on the critical Physics concepts and help the student solve the current problem.

Generating Feedback

Once the errors in a student's expression close to the right answer have been localized and identified, the next step is to generate useful feedback. The feedback is based in part on:

- the particular error,
- whether the terms in the answer are Vector or scalar quantities *and*
- the particular property (key) used to identify objects in the equation

This section describes some of the errors that are detected and the particular message that is generated for the student. All examples are based on the "block on inclined plane" problem where the terms are Forces (Vector) and the key for each term is Mass. Note that the error categories are generic to algebraic equations while the feedback is specific to Physics problems. As a reminder the correct answer is displayed in Figure 2.

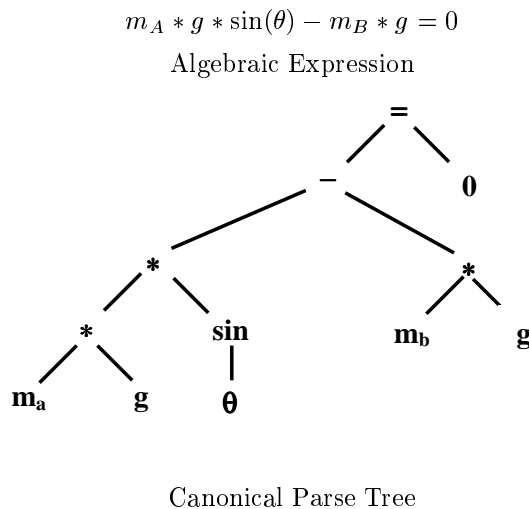


Figure 2: Correct Solution: Expression and Parse Tree

One of the many classes of wrong answers are those with an extra expression in the equation. The expression may be a term or a factor and it may, or may not, have an interpretation with specific relevance to Physics. This example includes an extra term that has an interpretation with specific relevance to Physics. An example of a wrong answer to this problem is provided in Figure 3.

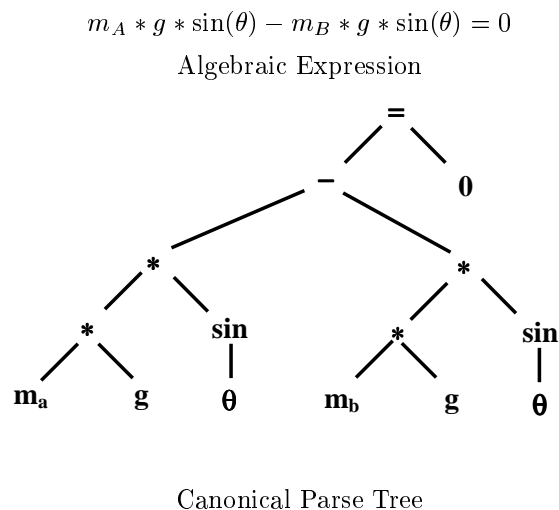
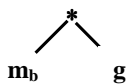
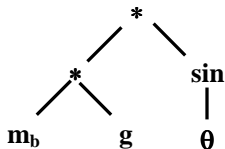


Figure 3: Incorrect Answer: Expression and Parse Tree

The algebraic expression may be provided in any form equivalent to that shown in Figure 3. The PHYSICS-TUTOR system will apply the necessary transformation to construct a canonical parse tree and compare that tree to the parse tree of the correct answer. When the supplied answer is correct this is a straight-forward match. When the answer is incorrect, as in this case, the system attempts to find the *best* match (*i.e.*, the one with the fewest mismatches) thereby focusing on the components of the two trees that differ. In this example the system reduces the problem to a mismatch of the subtrees shown in Figure 4.



Component of Parse Tree from Correct Answer



Component of Parse Tree from Incorrect Answer

Figure 4: Mismatched Components

Once minimal mismatch has been identified, the PHYSICS-TUTOR system attempts to classify the er-

ror. In this case it determines that an extra factor is present.

To this point, the process of localizing the error has relied entirely on algebraic transformations. Once the type of error (*i.e.*, an extra term of $\sin(\theta)$) is identified, the system applies Physics knowledge to further refine its reply.

In this case the extra term is a trigonometric function applied to a force vector. For the statics component of a Physics course the application of a trigonometric function to a vector indicates that the vector is being projected. The PHYSICS-TUTOR system combines this knowledge of Physics with the algebraic mismatch detected and replies as required. In this example a student input of

$$m_A * g * \sin(\theta) - m_B * g * \sin(\theta) = 0$$

is replied to by the PHYSICS-TUTOR system with the following hint:

“The Force on the object with Mass m_B is projected incorrectly”.

As this and the following examples show, our algorithm merges identification of the conceptual error and generation of an appropriate message into a single step.

- extra term: $m_A * g * \sin(\theta) - m_B * g + m_C * g = 0$
“Think about how the Force $m_C * g$ is involved in this equation”
- missing term: $m_B * g = 0$
“There is another Force in this equation. It is associated with the object with Mass m_A ”
- missing factor: $m_A * g - m_B * g = 0$
“The Force on the object with Mass m_A must be projected.”
- incorrect projection: $m_A * g * \cos \theta - m_B * g = 0$
“The Force from $m_A * g * \cos(\theta)$ is projected incorrectly”

Domain Knowledge

What kind of knowledge do we need in order to provide useful feedback for these kinds of problems? How much knowledge do we need? The PHYSICS-TUTOR system contains the following domain knowledge:

- knowledge about algebraic equations and the trigonometry functions: This knowledge is used to manipulate algebraic expressions and to transform them into a canonical form. The knowledge of trigonometry is also used to differentiate between an

incorrect factor (*e.g.*, a instead of g for acceleration) and an incorrect projection (*e.g.*, \cos in place of \sin).

- basic knowledge about Physics: This knowledge includes distinguishing between vector and scalar quantities, recognizing properties (*e.g.*, dimensions) of objects, and knowing the relationship, as defined in Physics, among properties such as force, mass, and velocity. This knowledge is used to generate the appropriate feedback in Physics terms rather than algebraic terms. Some of this knowledge was described in our earlier paper that focussed on how we could make sense of the terms in an algebraic equation.

Conclusion

This paper has described how the PHYSICS-TUTOR system handles the issues of (1) identification of errors and (2) generation of useful feedback in the context of a tutoring system for an introductory college level Physics course. We have identified the specific algebraic errors that can be made by the student and have described how domain knowledge is used to generate useful feedback in response to such errors. We are currently evaluating our system on other Physics concepts and problems to determine the generality of our approach.

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