Physics 618 Eighteenth Lecture March 28, 2017

Last time we completed our investigation of the machinery for finding irreducible representations of SU(N) and SO(N) in terms of direct products of the defining representation. We now know they are characterized by Young graphs and have a magic prescription for determining the dimensions.

Today

Today we will complete our discussion of representations of SU(N), learning how to decompose the tensor product of two irreducible representations. This is especially important in the scattering of particles, because if the scattering matrix is SU(3) invariant, the scattering amplitudes can be decomposed using the Wigner-Eckhart theorem into a smaller number of reduced matrix elements. But we will find a complication which did not arise in SU(2). The decompositions and corresponding Clebsch-Gordon (physics version) coefficients are also important in understanding the partial decay rates of hadronic resonances.

The tensor products also occur when considering that the quantum state depends on several variables for each of the components. For example, a baryon consisting of three quarks depends on the color, flavor, and spin of each of the three quarks, as well a possibly on the orbital angular momentum. We shall see that this gives good agreement with observed light baryons only if color is taken into effect.

Up to this point we have been considering symmetries which act on the entire configuration space uniformly, so the states of the system are transformed by one single group transformation. We are now about to make a big transition in how to apply group theory to physics. The discrete symmetry transformations we have considered, largely in classifying states of a system, are very useful, but not nearly as sexy as the idea of gauge symmetry groups, which we now understand as underlying all of fundamental physics. The basic idea is that physical degrees of freedom are local, described by fields (in the physicists sense of functions on space-time) which have symmetry properties independently at each space-time point, a local symmetry.

I need to ask you about your background, because my guess is that you are not all familiar with the dynamics of fields, in particular the lagrangian mechanics of distributed degrees of freedom. If that is correct, I will first describe the dynamics of a massless string under tension with a large number of point masses on it, for which you know how to apply ordinary lagrangian mechanics, and take a continuum limit of this to get the lagrangian approach to a massive string, the simplest (one-dimensional) field theory. Then we will expand to three spacial dimensions and see that time comes in naturally as a fourth component. My lecture notes for this are added ad-hoc from my lecture notes for classical mechanics, Physics 507, which you can find in full at

http://www.physics.rutgers.edu/~shapiro/507/gettext.shtml[1]

but I have posted an extraction from that which should be all you need for the next lecture. If we take the local degrees of freedom to be a set of scalars η_i, and the Lagrangian density to be $\mathcal{L} = \frac{1}{2}\sum_i \left(\sum_\mu (\partial_\mu \eta_i)(\partial^\mu \eta_i) + m^2\eta_i^2\right)$ we get the equations of motion $\left(\partial_t^2 - \nabla^2 - m^2\right)\eta_i = 0$, the Klein-Gordon equation for a free particle of mass m (with $\hbar = c = 1$).

Of course if you are all well familiar with the lagrangian formulation of fields, we can skip this chapter and go directly to Local Symmetry. That will ask how we can make a theory invariant under independent symmetry transformations at each point in space-time. This will lead us to Gauge Field Theory.

Reminder:

Seventh Homework due Thursday, March 30 at 4:00 PM
Eighth Homework is now posted, due April 6 at 4:00 PM

To discuss: Do we need an introduction to field theory?

[1]In particular, pp 136-150 and 233-260.