Physics 618 Twelfth Lecture Feb. 24, 2017

Last time we began our discussion of compact semisimple Lie algebras beyond SU(2), starting by forming a basis with the m vectors H_i which are a basis of the Cartan subalgebra. We started analyzing their arbitrary representations $|D\rangle$ by diagonalizing the H_i, and defining their eigenvalues $\vec{\mu}$ which are the **weights** of the representation. In particular, for the adjoint representation, the weights for all the other dimensions of the of the Lie algebra $\mathcal{L}$, $\vec{\alpha}_j$, are called the roots and the corresponding algebra elements $E_{\pm\vec{\alpha}_j}$ are raising and lowering (in the $\vec{\alpha}$ direction) operators. We found the "master formula" for the weights,

$$\frac{\vec{\mu}\cdot\vec{\alpha}}{\alpha^2} = \frac{q-p}{2},$$

and applying this to the adjoint representation, we found that any two roots satisfy $\dfrac{\vec{\alpha}\cdot\vec{\beta}}{\alpha^2} = \dfrac{n}{2}$ for $n \in \mathbb{Z}$, which restricted the angles between weights to just a few possibilities, and also the ratio of lengths. We also showed that no two roots are in the same direction.

We then considered the group SU(3), which has played an important role historically for flavor in particle physics in the 60's, but plays a fundamental role for color in QCD. We see that the root vectors are multiples of 60° apart.

Today

We will see that this constraint is extremely powerful, so that within a week we will have found all the possible semisimple finite-dimensional compact Lie groups. We begin by defining an (arbitrary) ordering on the roots and defining the **simple** roots, whose action on the representation is enough to determine everything. There are m simple roots, where m is the rank, and we will see that they are always at least 90° apart. Together with the master formula this will restrict simple compact Lie algebras to 4 countably infinite classes and five other algebras.

Reminders:

There will be a midterm exam on Tuesday, March 7. I currently think this will cover the material from the beginning through Dynkin diagrams (Chapter 8), and possibly fundamental weights (Chapter 9). It will be closed book, but you are allowed two sheets of $8\,\frac{1}{2} \times 11$ inch handwritten notes.

Homework 6 will be due Thursday, March 2. It has been posted.