

# Group Theory in Physics

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Note that these are lecture notes, not a publication, and the author acknowledges that much of this material is taken from the books of Joshi, Jones, and Georgi.

The author welcomes corrections, comments, and criticism.

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